MODELLING OF REPEATED CREEP AND RECOVERY FROM LIMITED DYNAMIC TESTING DATA

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$$G'(\omega) = G_e + \sum_{i=1}^{N} g_i \frac{(\omega\lambda_i)^2}{1+(\omega\lambda_i)^2}$$

$$G''(\omega) = \sum_{i=1}^{N} g_i \frac{\omega\lambda_i}{1+(\omega\lambda_i)^2}$$

$$G(t) \approx \sum_{i=1}^{\infty} g_i e^{-t/\lambda_i}$$

CREEP AND RECOVERY

$$\gamma(t) = \tau_0\left[J_g + t/\eta_0 + \sum_{i=1}^{N-1} J_i\left(1 - e^{-t/\Lambda_i}\right)\right]$$

$$\gamma(t) = \tau_0\left[t'/\eta_0 + \sum_{i=1}^{N-1} J_i\left(e^{-(t-t')/\Lambda_i} - e^{-t/\Lambda_i}\right)\right]$$

$$J(t) = \gamma(t)/\tau_0$$

THE INPUT SHEAR-STRESS

$$\tau (t) = \tau_0 [U (t - 10n) - U (t - 10n - 1)]$$

where τ_0 = const. (300 Pa, in our case), $n = 1, 2, 3, \ldots$ and U is the Heaviside step function ($U(t) = 0$, for $t < 0$, and $U(t) = 1$, for $t > 0$)

Then in the n-th cycle ($n = 1, 2, 3, \ldots$)
we have

Creep, $10(n - 1) < t < 10(n - 1) + 1$

$$J(t) = (n-1)A + \frac{t-10(n-1)}{\eta_0} + J_g + \sum_{i+1}^{N-1} J_i \left(1 - e^{-\frac{t-10(n-1)}{\Lambda_i}}\right)$$

Recovery,

$10(n - 1) + 1 < t < 10(n - 1) + 10$

$$J(t) = (n-1)A + \frac{1}{\eta_0} + \sum_{i+1}^{N-1} J_i \left(1 - e^{-\frac{t-10(n-1)}{\Lambda_i}} \right)$$

where

$$A = 1/\eta_0 + \sum_{i=1}^{N-1} J_i \left(e^{-\frac{9}{\Lambda_i}} - e^{-\frac{10}{\Lambda_i}} \right)$$

Time Scaling

Creep

$$J(t) = J_g + t/c\eta_0 + \sum_{i=1}^{N-1} J_i \left(1 - e^{-\frac{t}{c\Lambda_i}}\right)$$

Recovery

$$J(t) = 1/\eta_0 c + \sum_{i=1}^{N-1} J_i \left(e^{-\frac{t-1}{c\Lambda_i}} - e^{-\frac{t}{c\Lambda_i}}\right)$$

Creep, $10(n - 1) < t < 10(n - 1) + 1$

$$J(t) = (n-1)A + \frac{t-10(n-1)}{\eta_0 c} + J_g + \sum_{i=1}^{N-1} J_i \left(1 - e^{-\frac{t-10(n-1)}{c\Lambda_i}} \right)$$

Recovery,

$$10(n - 1) + 1 < t < 10(n - 1) + 10$$

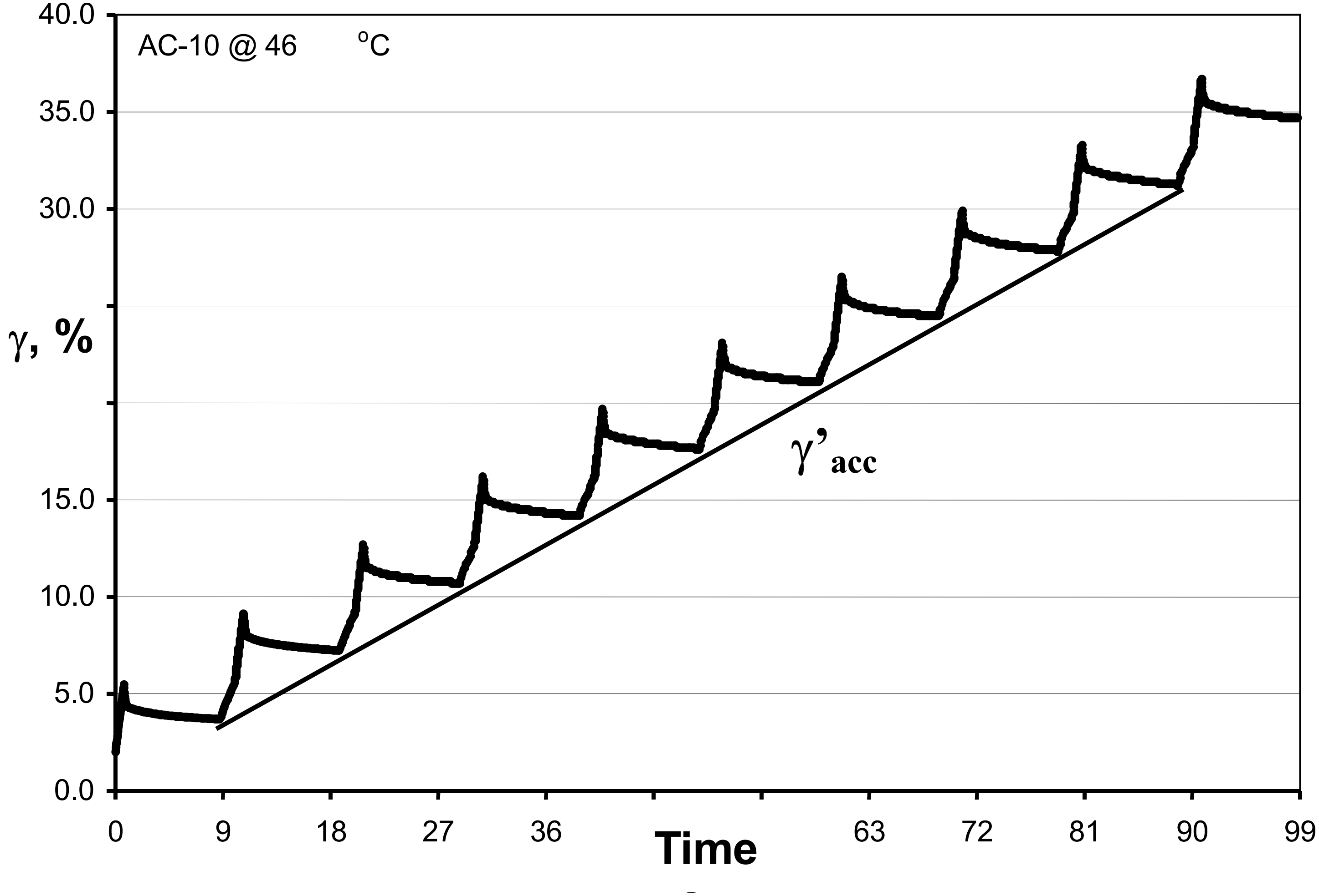
$$J(t) = (n-1)A + 1/\eta_0 c + \sum_{i+1}^{N-1} J_i \left(-e^{-\frac{t-10(n-1)-1}{c\Lambda_i}} - e^{-\frac{t-10(n-1)}{c\Lambda_i}} \right)$$

where

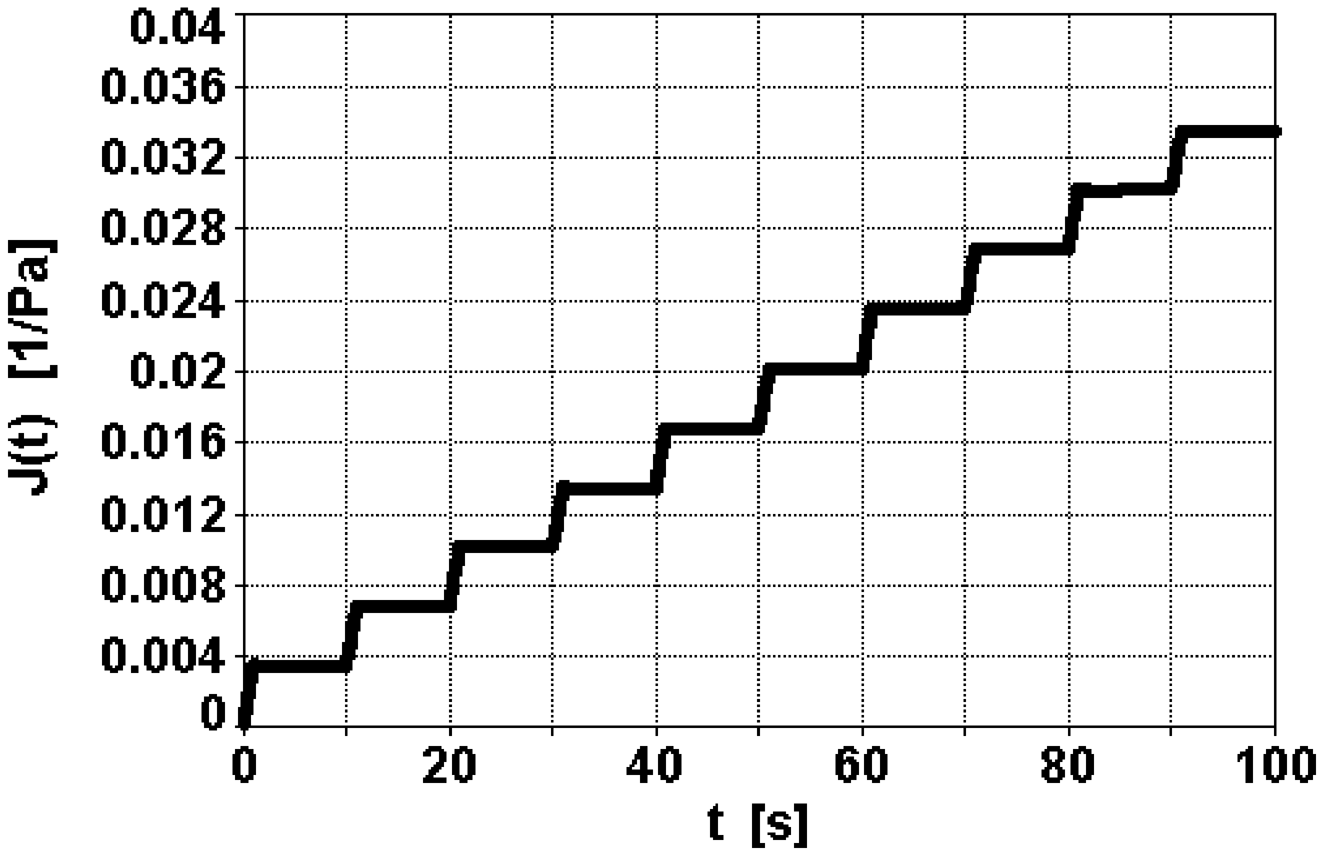
$$A = 1/\eta_0 c + \sum_{i=1}^{N-1} J_i \left(e^{-\frac{9}{c\Lambda_i}} - e^{-\frac{10}{c\Lambda_i}} \right)$$

TABLE 1. ASPHALT BINDERS EVALUATED IN THIS STUDY

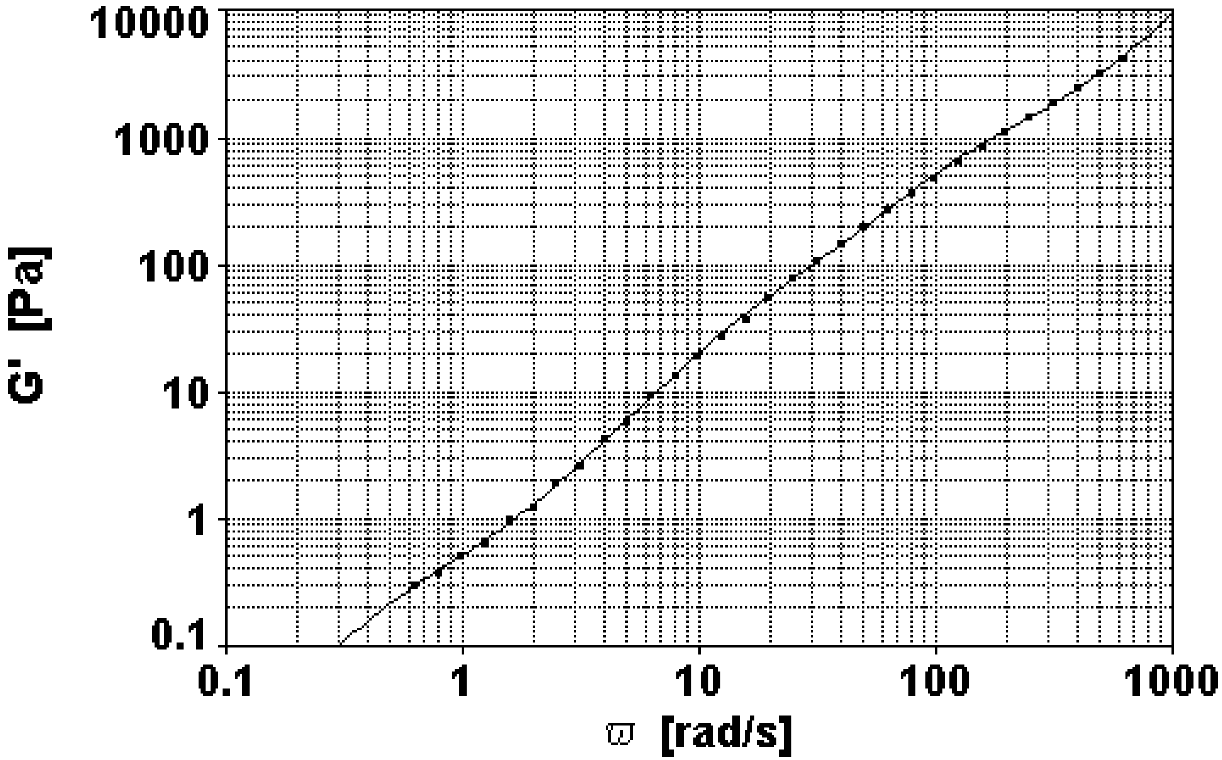
Asphalt Binder	Performance History Available
PG64-22- Conventional	Nevada D.O.T
AC20P-PMA	Nevada D.O.T
Cariphalte DM-PMA	Anecdotal in Europe



40.0
35.0
30.0
15.0
10.0
5.0
0.0
AC-10 @ 46 °C
γ, %
γ'acc
0
9
18
27
36
63
72
81
90
99
Time



Creep compliance J(t), T = 64° C. First ten cycles. Nevada 64-22

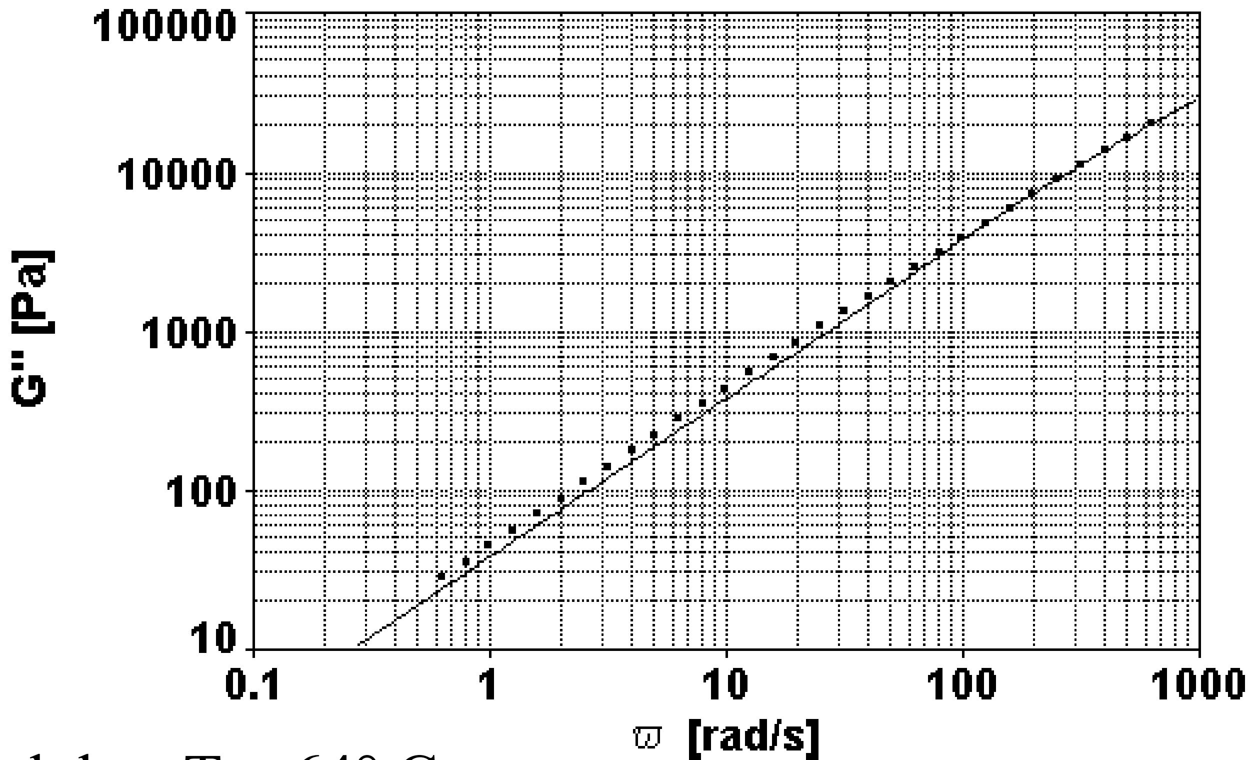


Storage modulus, T = 64° C.

■ experimental,

— calculated from the discrete relaxation spectrum (4 modes).

Nevada 64-22.

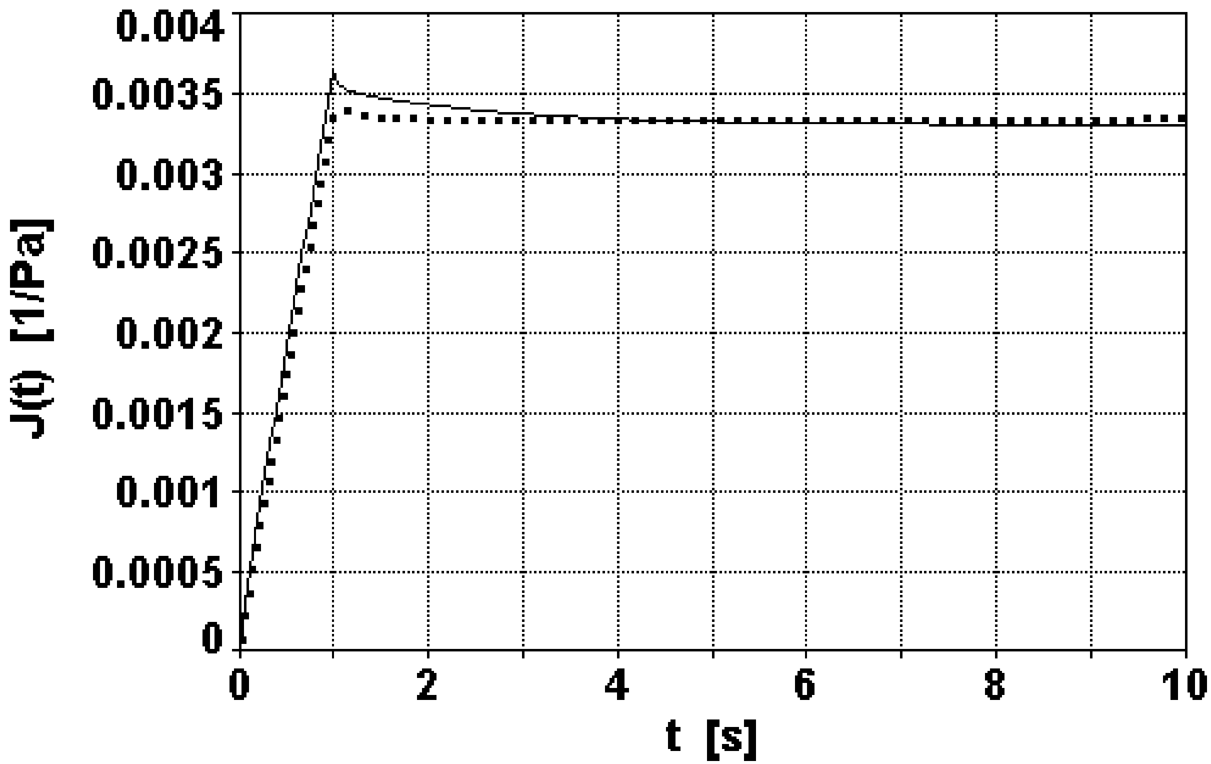


Loss modulus, T = 64° C.

■ experimental,

▭ calculated from the discrete relaxation spectrum (4 modes).

Nevada 64-22.

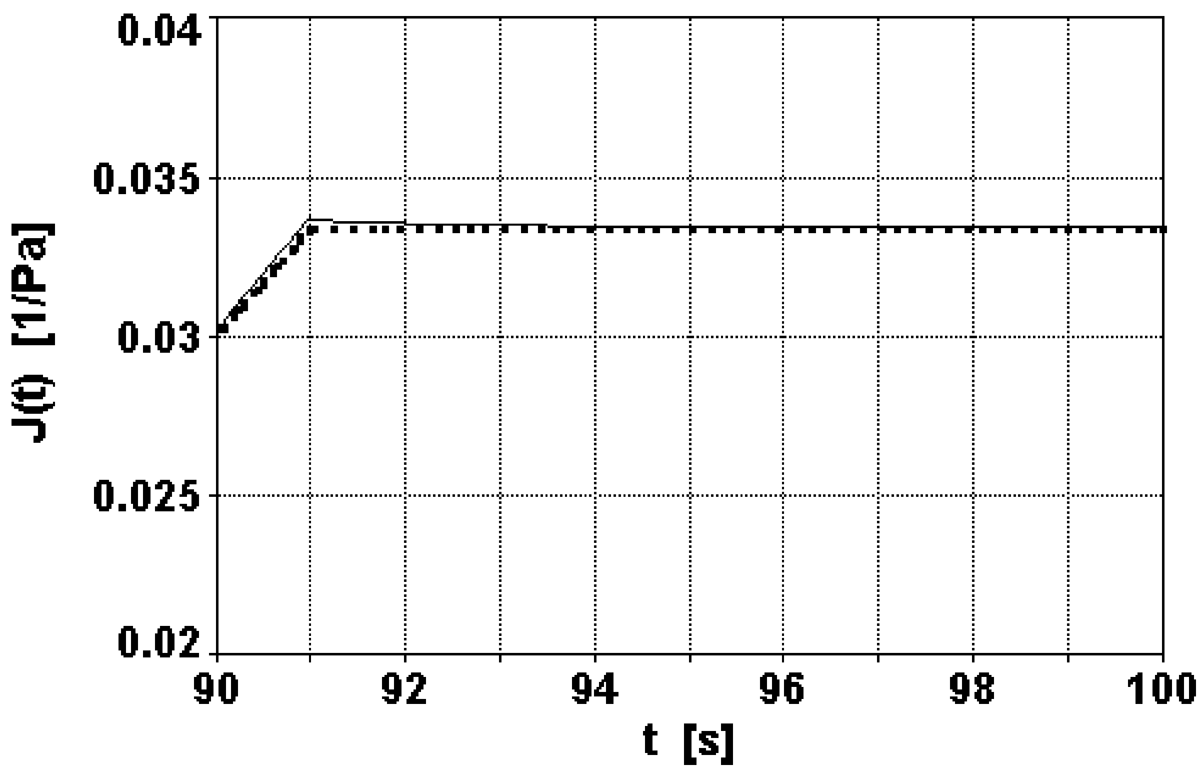


First cycle, T = 64° C.

■ experimental,

calculated from Equs. (13)-15).

Nevada 64-22.

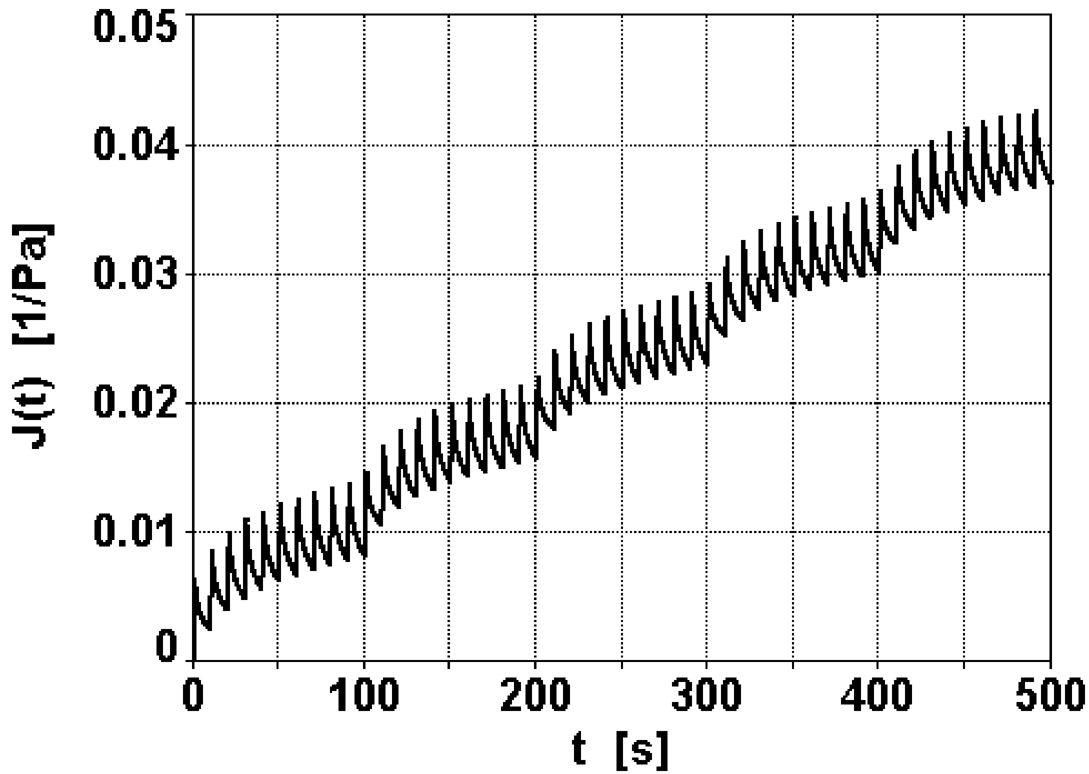


Cycle n=10, T = 64° C.

■ experimental,

calculated from Equs. (13)-(15).

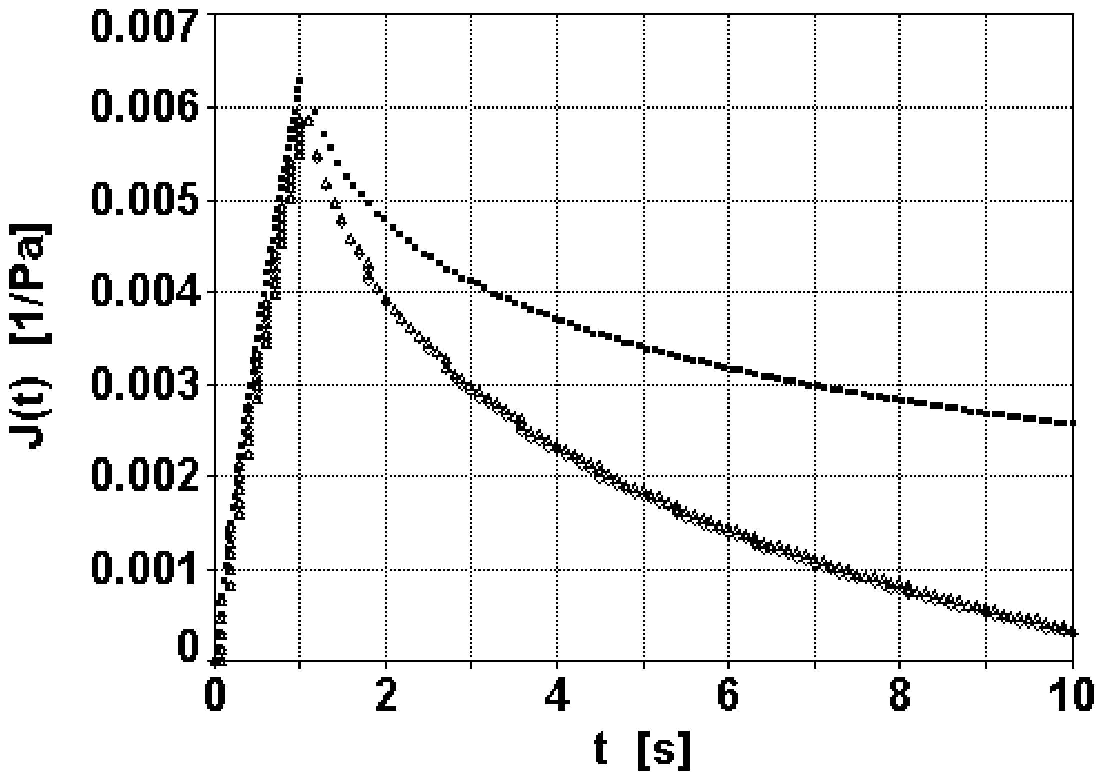
Nevada 64-22.



Creep compliance J(t),

T = 64° C. First fifty cycles.

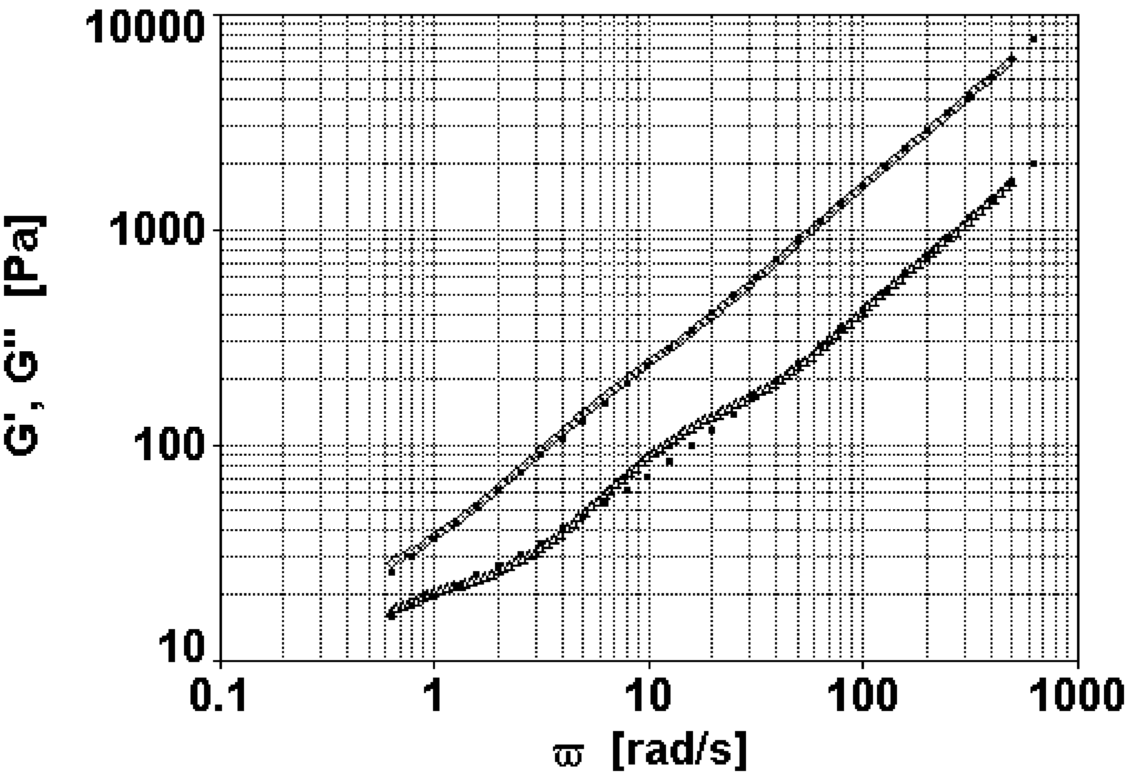
Nevada AC20P.



Creep compliance J(t), T = 64°C.

cycle 1, Δ cycle 10, ◊ cycle 50.

Nevada AC20P.

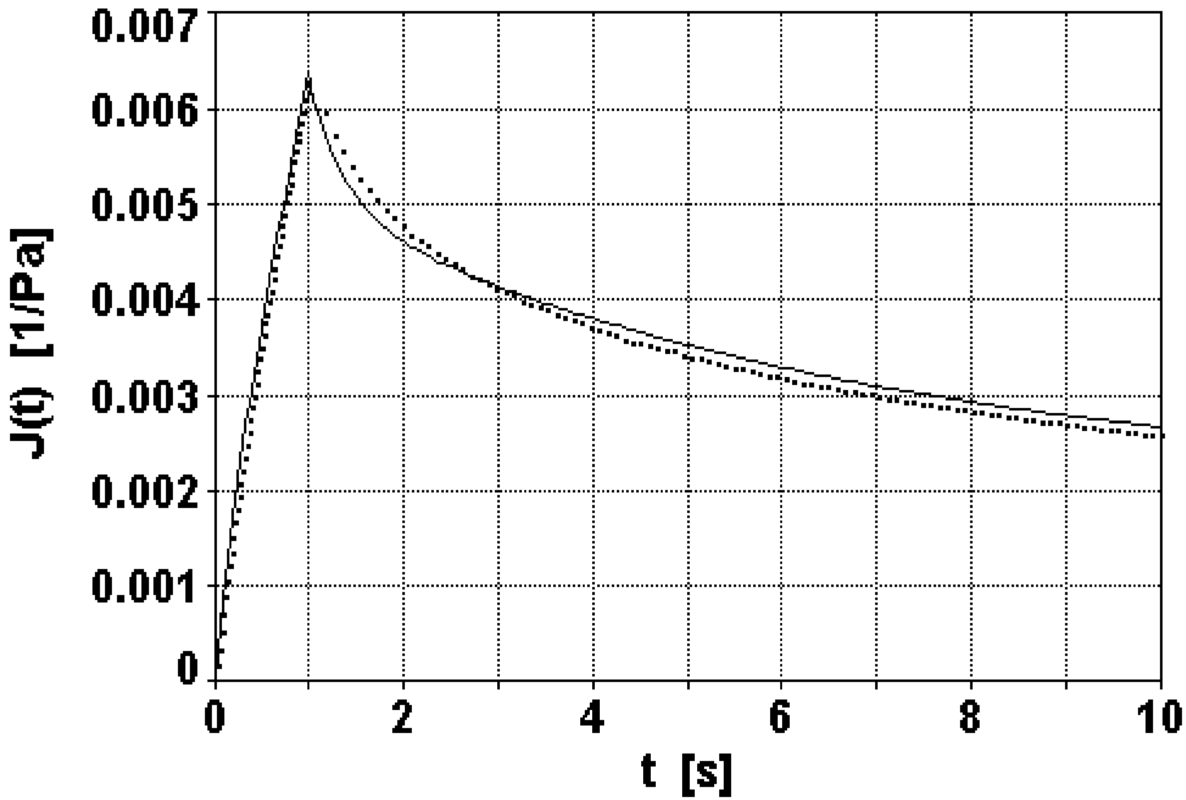


Dynamic moduli, T =64C.

Experimental: Δ G', ◊ G'',

■ calculated from the discrete relaxation spectrum (5 modes).

Nevada AC 20P.

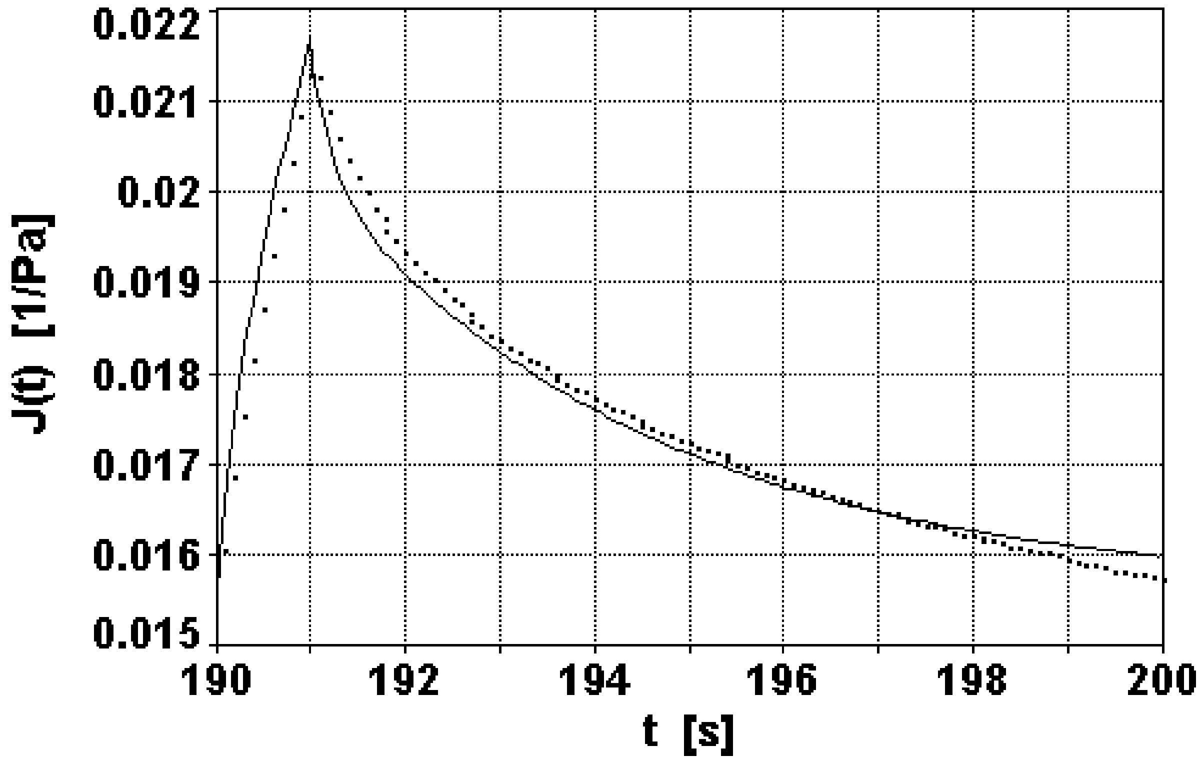


First cycle - repeated creep and recovery test, T = 64° C.

■ experimental,

calculated from Equs. (13)-(15).

Nevada AC20P.

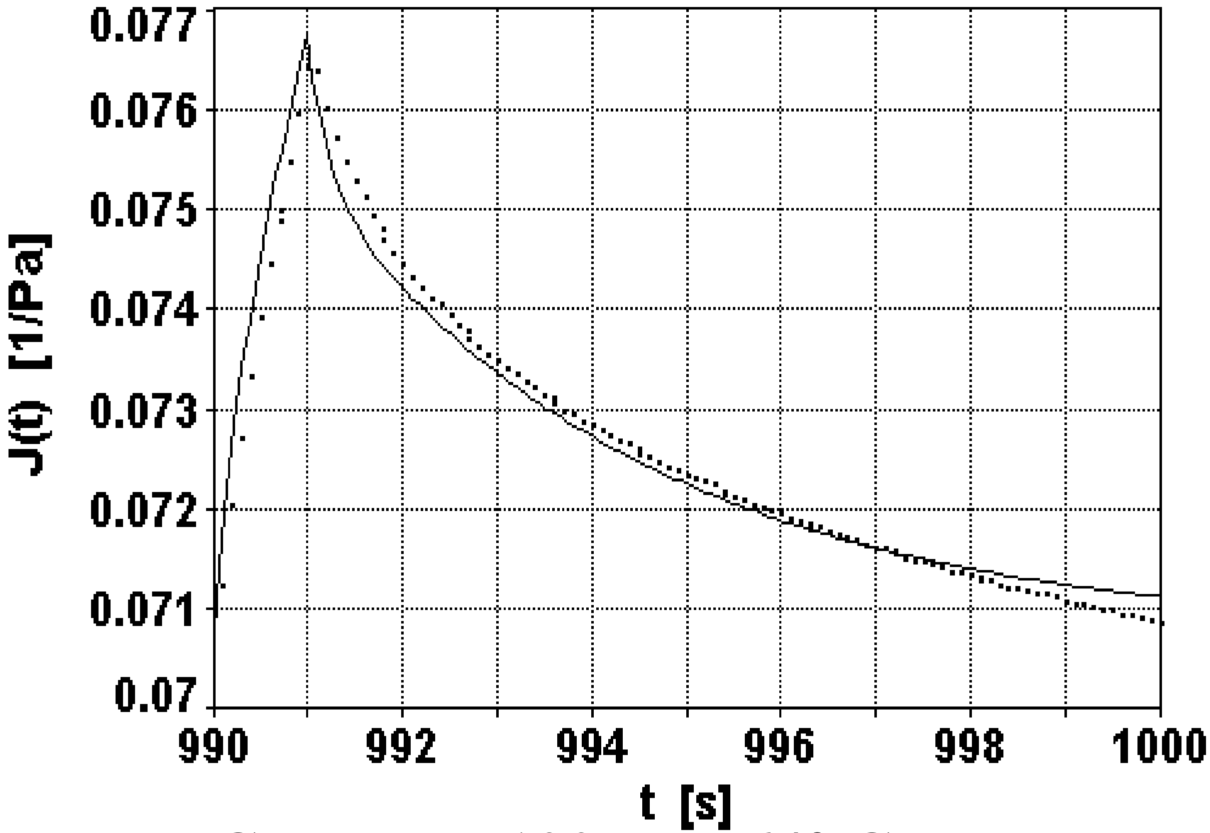


Cycle, n = 50, T = 64° C.

■ experimental,

calculated from Equs. (13)-(15).

Nevada AC20P.

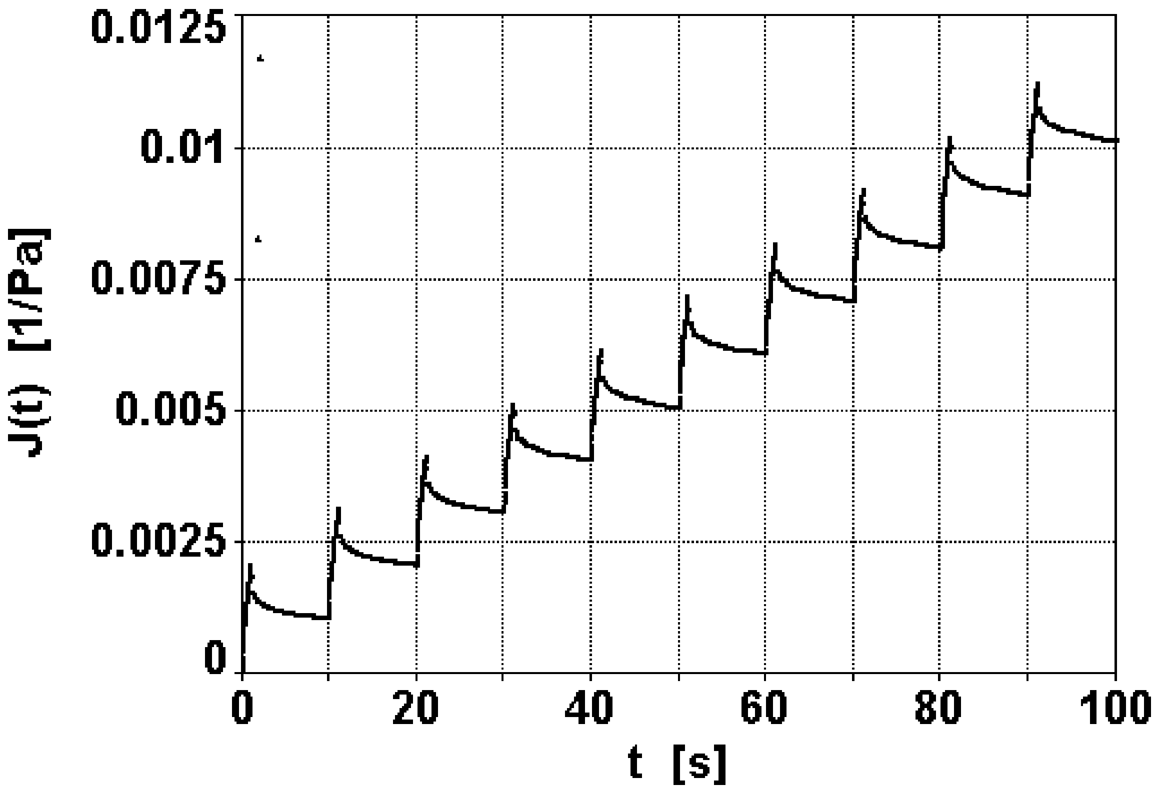


Cycle, n = 100, T = 64° C.

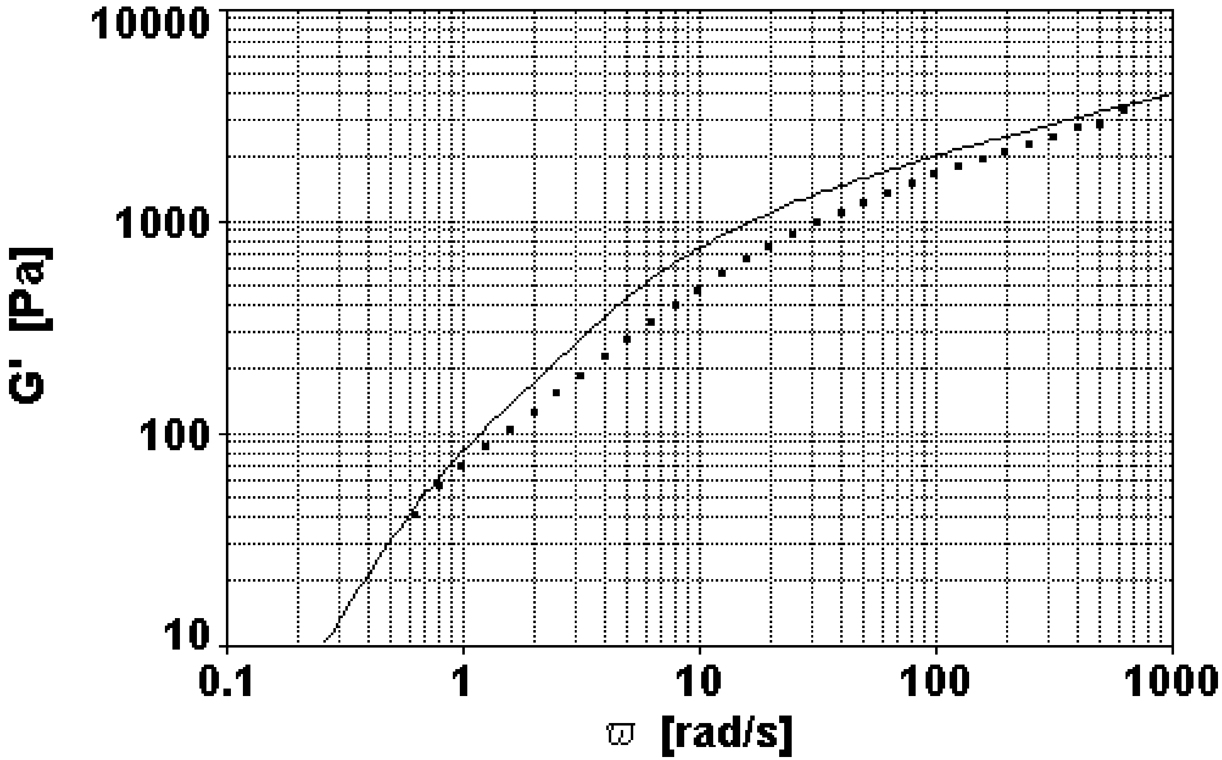
■ experimental,

calculated from Equs. (13)-(15).

Nevada AC20P.



Creep compliance J(t), T = 76° C. First ten cycles. Cariphalt 76.

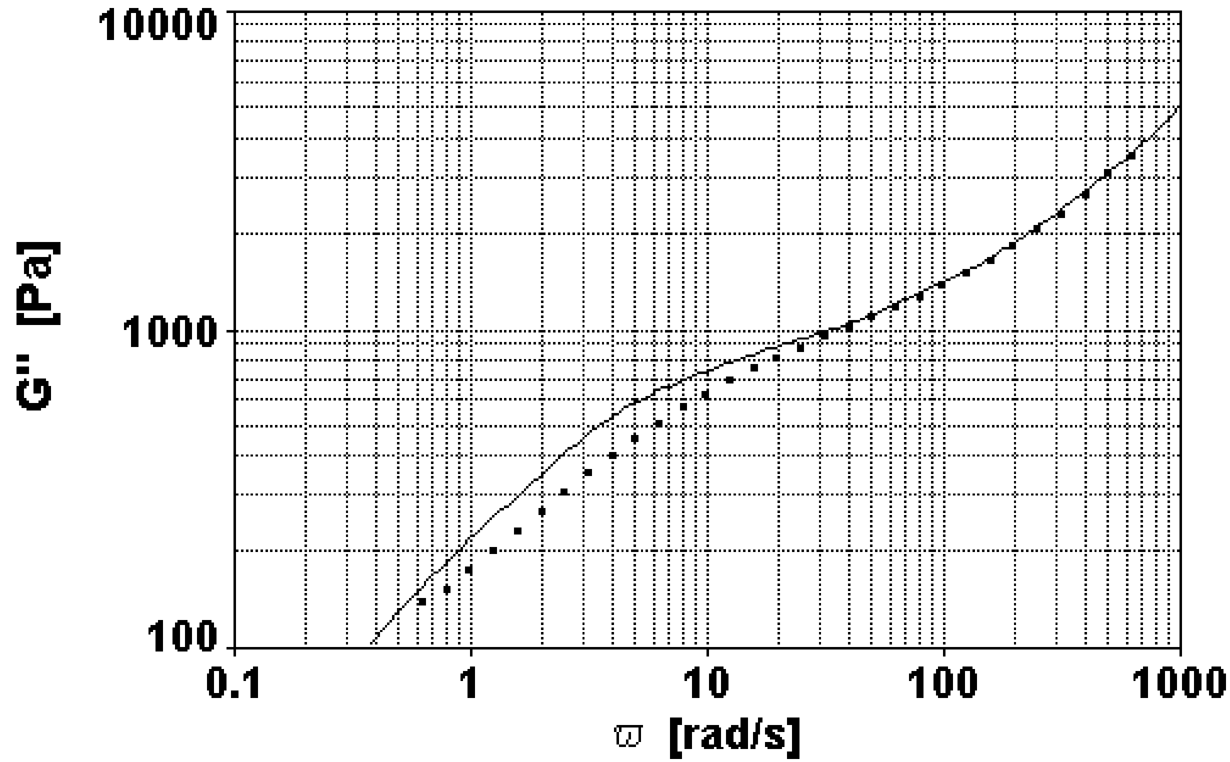


Storage modulus, T = 76° C.

■ experimental,

▭ calculated from the discrete relaxation spectrum (6 modes).

Cariphalt 76.

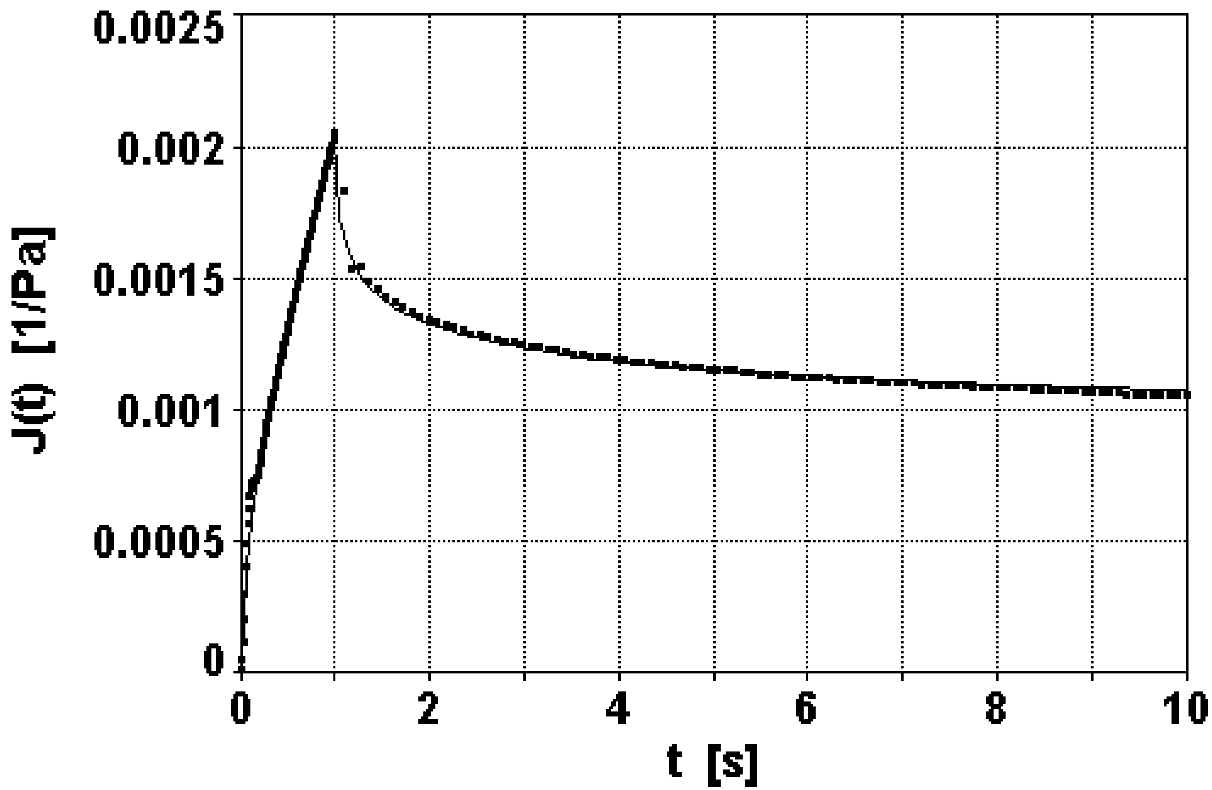


Loss modulus, T = 76° C.

■ experimental,

calculated from the discrete relaxation spectrum (6 modes).

Cariphalt 76.

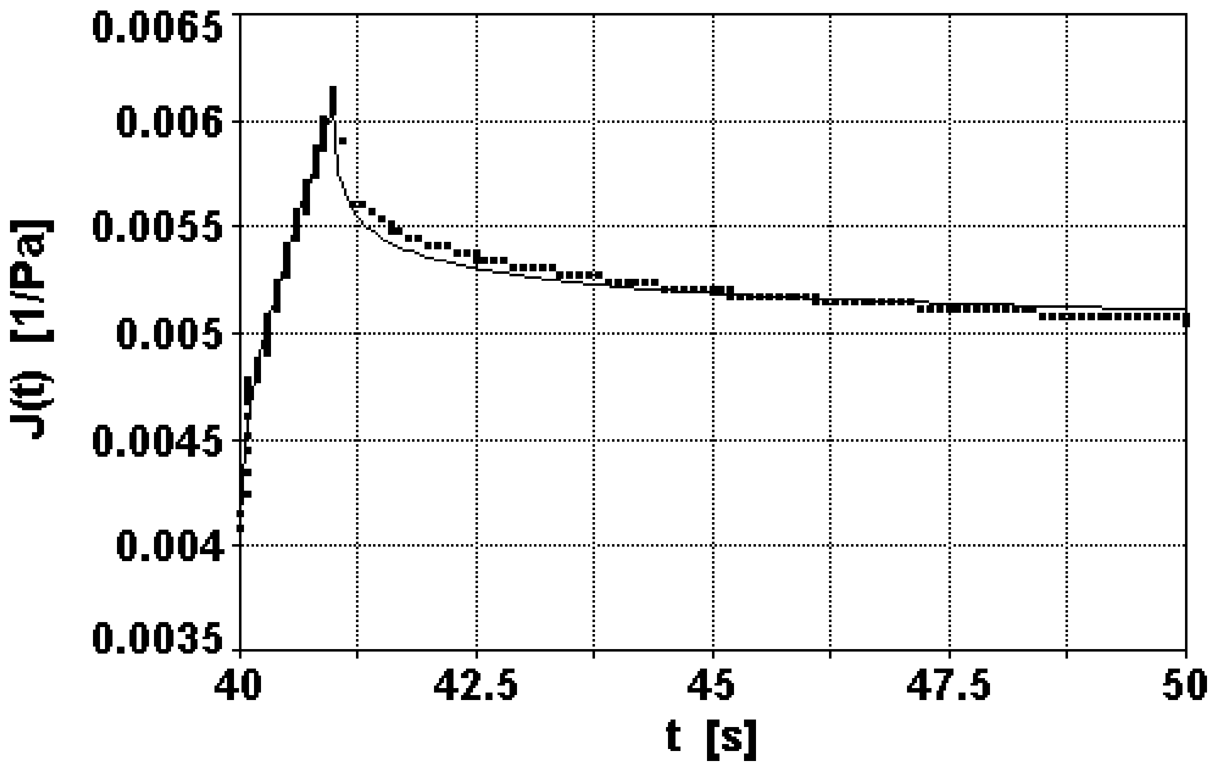


Creep compliance J(t), T = 76° C. First cycle,

■ experimental,

calculated from Equs. (13)-(15).

Cariphalt 76.

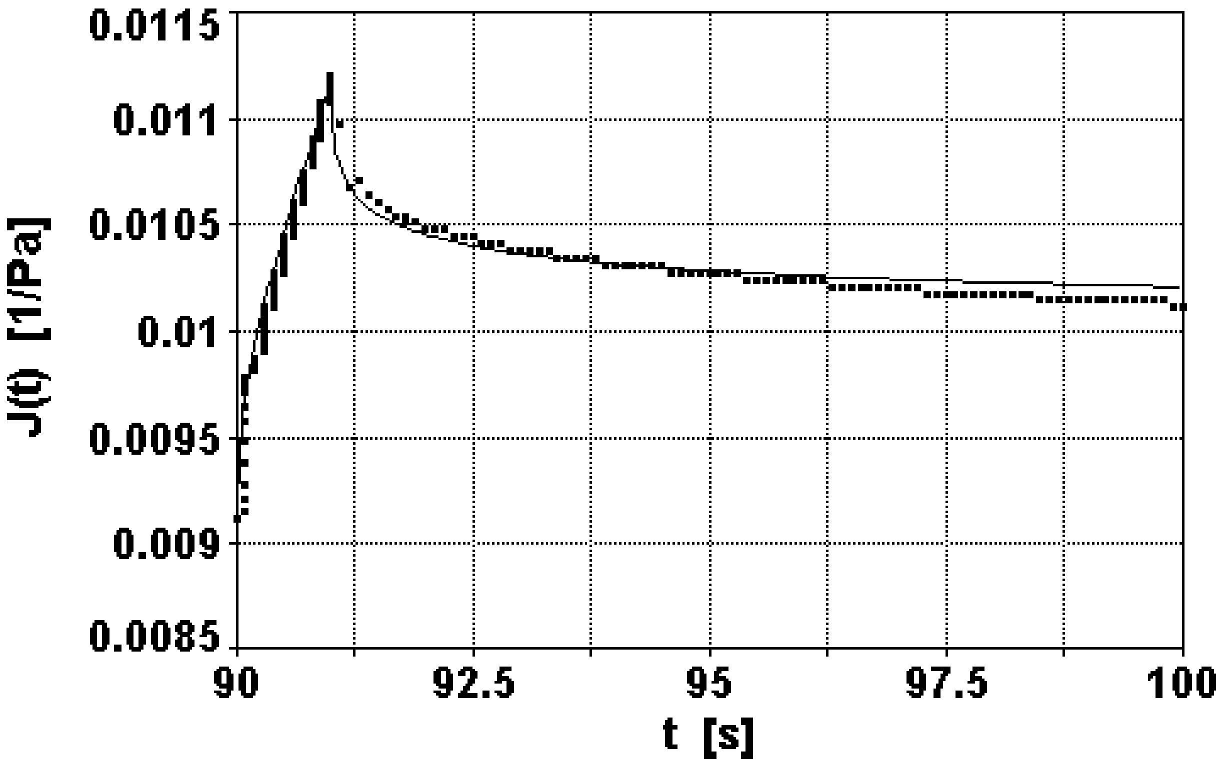


Creep compliance J(t), T = 76° C. Fifth cycle,

■ experimental,

calculated from Equs.(13)-(15).

Cariphalt 76.



Creep compliance J(t), T = 76° C. Cycle n = 10,

■ experimental,

— calculated from Equs. (13)-(15).

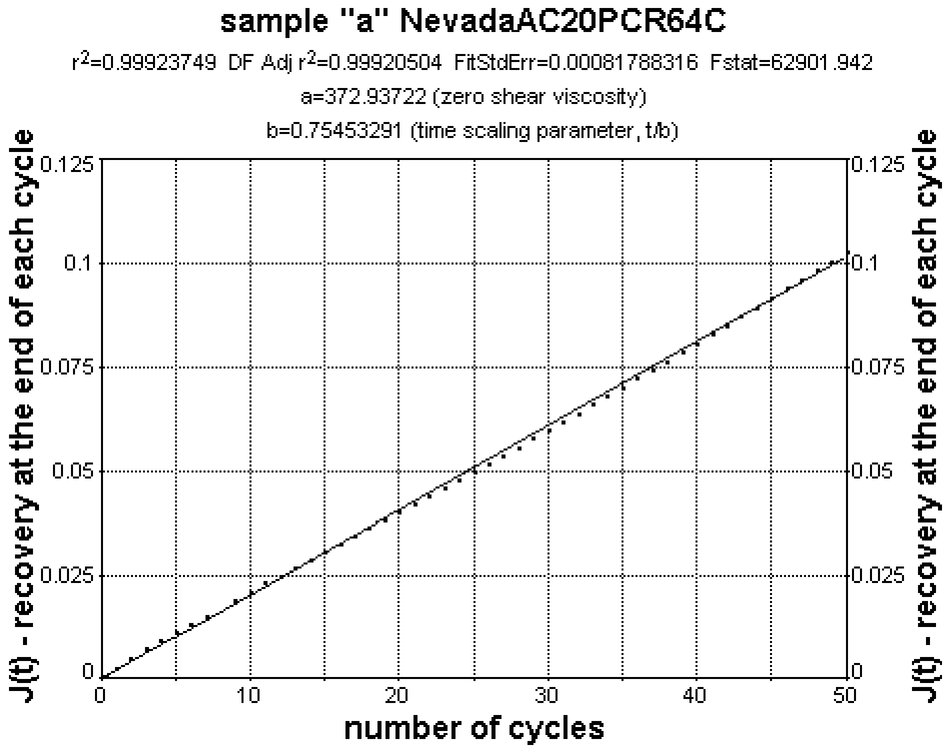
Cariphalt 76.

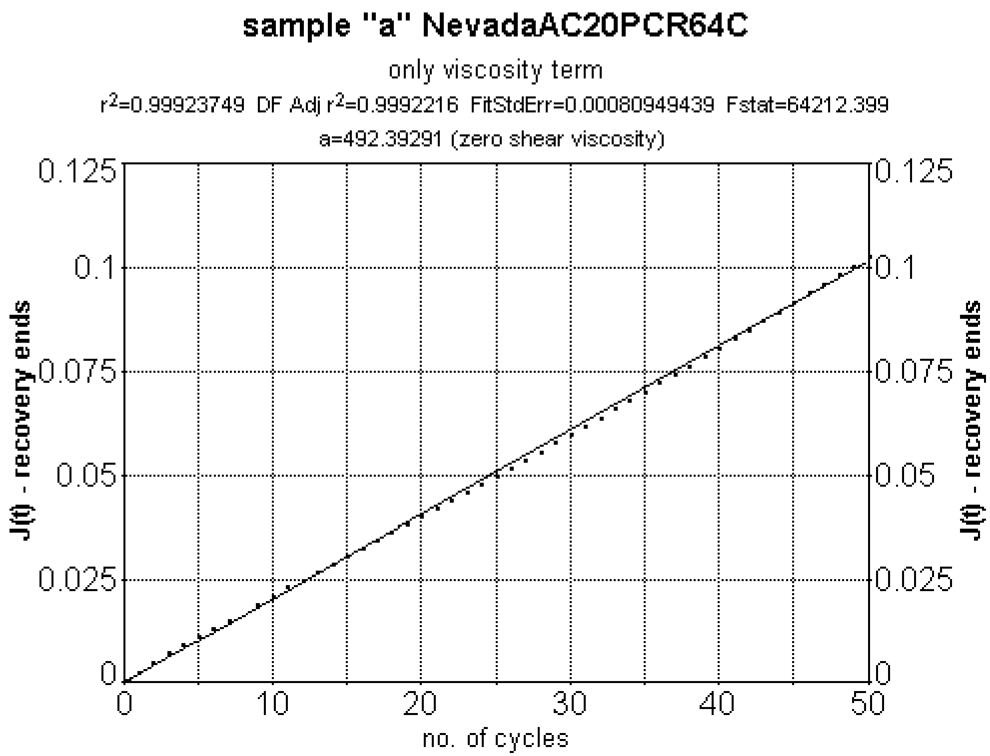
CONCLUSION

The Boltzmann superposition principle governs the repeated creep and recovery test in the linear viscoelastic domain of small deformations. For the test, set to perform creep during the first one second followed by nine seconds of recovery, and repeated n-times one has:

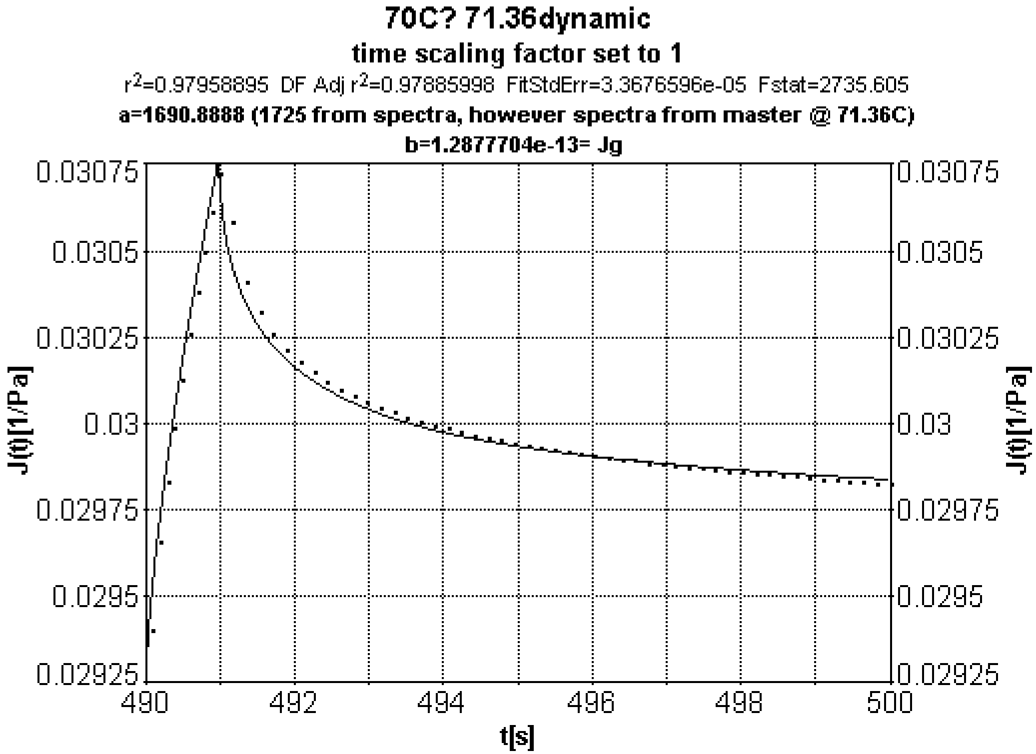
Creep $J(t) = J(t-10(n-1)) + (n-1)(J(10)-J(9)$

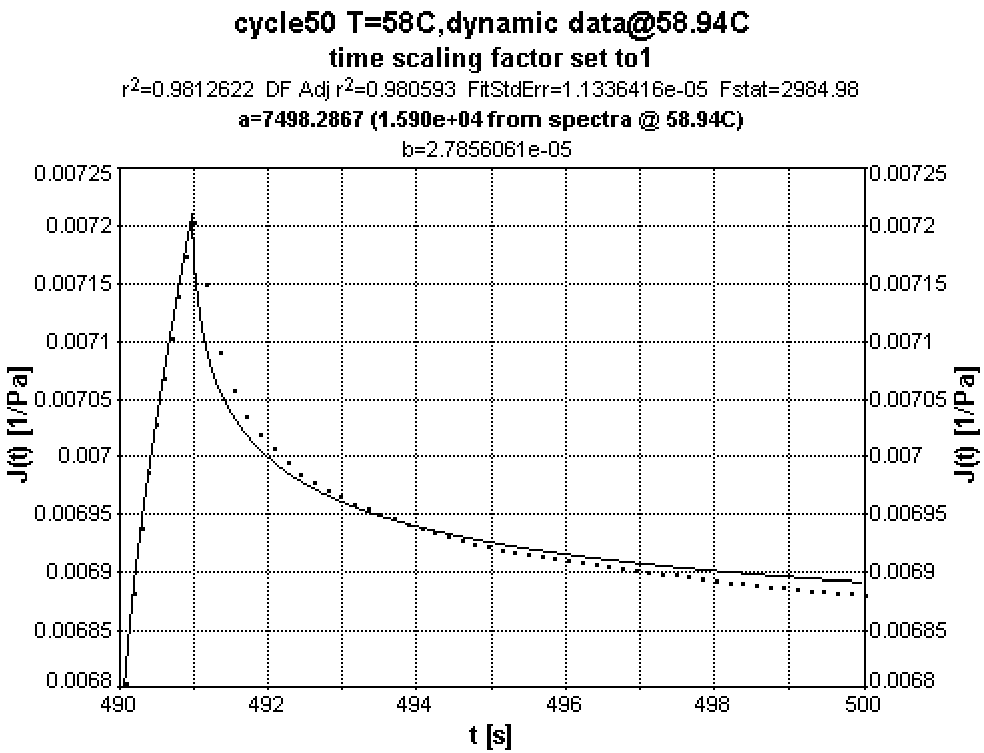
Recovery $J(t) = J(t-10(n-1))-J(t-10(n-1)-1) + (n-1)(J(10)-J(9))$

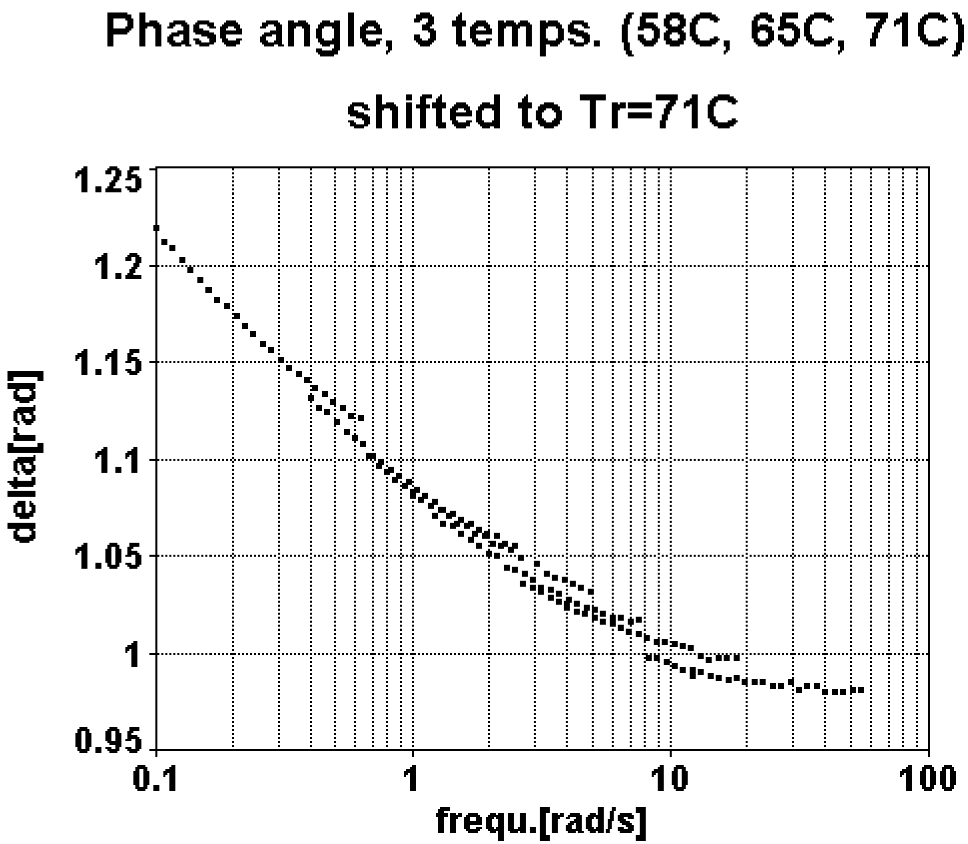

sample "a" NevadaAC20PCR64C
r2=0.99923749 DF Adj r2=0.99920504 FitStdErr=0.00081788316 Fstat=62901.942
a=372.93722 (zero shear viscosity)
b=0.75453291 (time scaling parameter, t/b)
J(t) - recovery at the end of each cycle
0.125
0.1
0.075
0.05
0.025
0
0
10
20
30
40
50
number of cycles



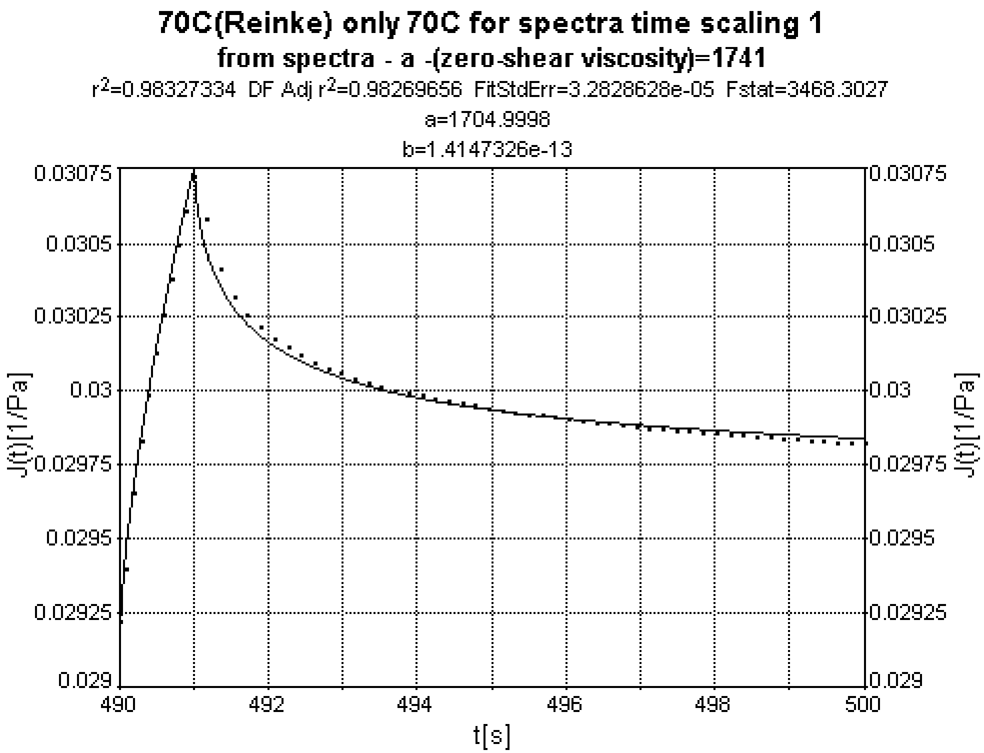
sample "a" NevadaAC20PCR64C
only viscosity term
r2=0.99923749 DF Adj r2=0.9992216 FitStdErr=0.00080949439 Fstat=64212.399
a=492.39291 (zero shear viscosity)
J(t) - recovery ends
no. of cycles
0.125
0.1
0.075
0.05
0.025
0
10
20
30
40
50


70C? 71.36dynamic
time scaling factor set to 1
r2=0.97958895 DF Adj r2=0.97885998 FitStdErr=3.3676596e-05 Fstat=2735.605
a=1690.8888 (1725 from spectra, however spectra from master @ 71.36C)
b=1.2877704e-13= Jg
0.03075
0.0305
0.03025
0.03
0.02975
0.0295
0.02925
J(t)[1/Pa]
490
492
494
496
498
500
t[s]


cycle50 T=58C,dynamic data@58.94C
time scaling factor set to1
r2=0.9812622 DF Adj r2=0.980593 FitStdErr=1.1336416e-05 Fstat=2984.98
a=7498.2867 (1.590e+04 from spectra @ 58.94C)
b=2.7856061e-05
J(t) [1/Pa]
t [s]
0.00725
0.0072
0.00715
0.0071
0.00705
0.007
0.00695
0.0069
0.00685
0.0068
490
492
494
496
498
500



Phase angle, 3 temps. (58C, 65C, 71C)
shifted to Tr=71C
delta[rad]
1.25
1.2
1.15
1.1
1.05
1
0.95
0.1
1
10
100
frequ.[rad/s]


70C(Reinke) only 70C for spectra time scaling 1
from spectra - a -(zero-shear viscosity)=1741
r2=0.98327334 DF Adj r2=0.98269656 FitStdErr=3.2828628e-05 Fstat=3468.3027
a=1704.9998
b=1.4147326e-13
0.03075
0.0305
0.03025
0.03
0.02975
0.0295
0.02925
0.029
J(t)[1/Pa]
490
492
494
496
498
500
t[s]