

MODELLING OF REPEATED CREEP AND RECOVERY FROM LIMITED DYNAMIC TESTING DATA

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$$G'(\omega) = G_e + \sum_{i=1}^N g_i \frac{(\omega\lambda_i)^2}{1 + (\omega\lambda_i)^2}$$

$$G''(\omega) = \sum_{i=1}^N g_i \frac{\omega\lambda_i}{1 + (\omega\lambda_i)^2}$$

$$G(t) \approx \sum_{i=1}^{\infty} g_i e^{-t/\lambda_i}$$

CREEP AND RECOVERY

$$\gamma(t) = \tau_0 \left[J_g + t/\eta_0 + \sum_{i=1}^{N-1} J_i \left(1 - e^{-t/\Lambda_i} \right) \right]$$

$$\gamma(t) = \tau_0 \left[t'/\eta_0 + \sum_{i=1}^{N-1} J_i \left(e^{-(t-t')/\Lambda_i} - e^{-t/\Lambda_i} \right) \right]$$

$$J(t) = \gamma(t)/\tau_0$$

THE INPUT SHEAR-STRESS

$$\tau(t) = \tau_0 [U(t - 10n) - U(t - 10n - 1)]$$

where $\tau_0 = \text{const.}$ (300 Pa, in our case),
 $n = 1, 2, 3, \dots$ and U is the Heaviside
step function ($U(t) = 0$, for $t < 0$, and
 $U(t) = 1$, for $t > 0$)

Then in the n-th cycle ($n = 1, 2, 3, \dots$)
we have

Creep, $10(n - 1) < t < 10(n - 1) + 1$

$$J(t) = (n-1)A + \frac{t-10(n-1)}{\eta_0} + J_g + \sum_{i=1}^{N-1} J_i \left(1 - e^{-\frac{t-10(n-1)}{\Lambda_i}} \right)$$

Recovery,

$$10(n - 1) + 1 < t < 10(n - 1) + 10$$

$$J(t) = (n - 1)A + \frac{1}{\eta_0} + \sum_{i=1}^{N-1} J_i \left(1 - e^{-\frac{t-10(n-1)}{\Lambda_i}} \right)$$

where

$$A = 1/\eta_0 + \sum_{i=1}^{N-1} J_i \left(e^{-\frac{9}{\Lambda_i}} - e^{-\frac{10}{\Lambda_i}} \right)$$

Time Scaling

Creep

$$J(t) = J_g + t/c\eta_0 + \sum_{i=1}^{N-1} J_i \left(1 - e^{-\frac{t}{c\Lambda_i}} \right)$$

Recovery

$$J(t) = 1/\eta_0 c + \sum_{i=1}^{N-1} J_i \left(e^{-\frac{t-1}{c\Lambda_i}} - e^{-\frac{t}{c\Lambda_i}} \right)$$

Creep, $10(n - 1) < t < 10(n - 1) + 1$

$$J(t) = (n-1)A + \frac{t - 10(n-1)}{\eta_0 c} + J_g + \sum_{i=1}^{N-1} J_i \left(1 - e^{-\frac{t - 10(n-1)}{c\Lambda_i}} \right)$$

Recovery,

$$10(n - 1) + 1 < t < 10(n - 1) + 10$$

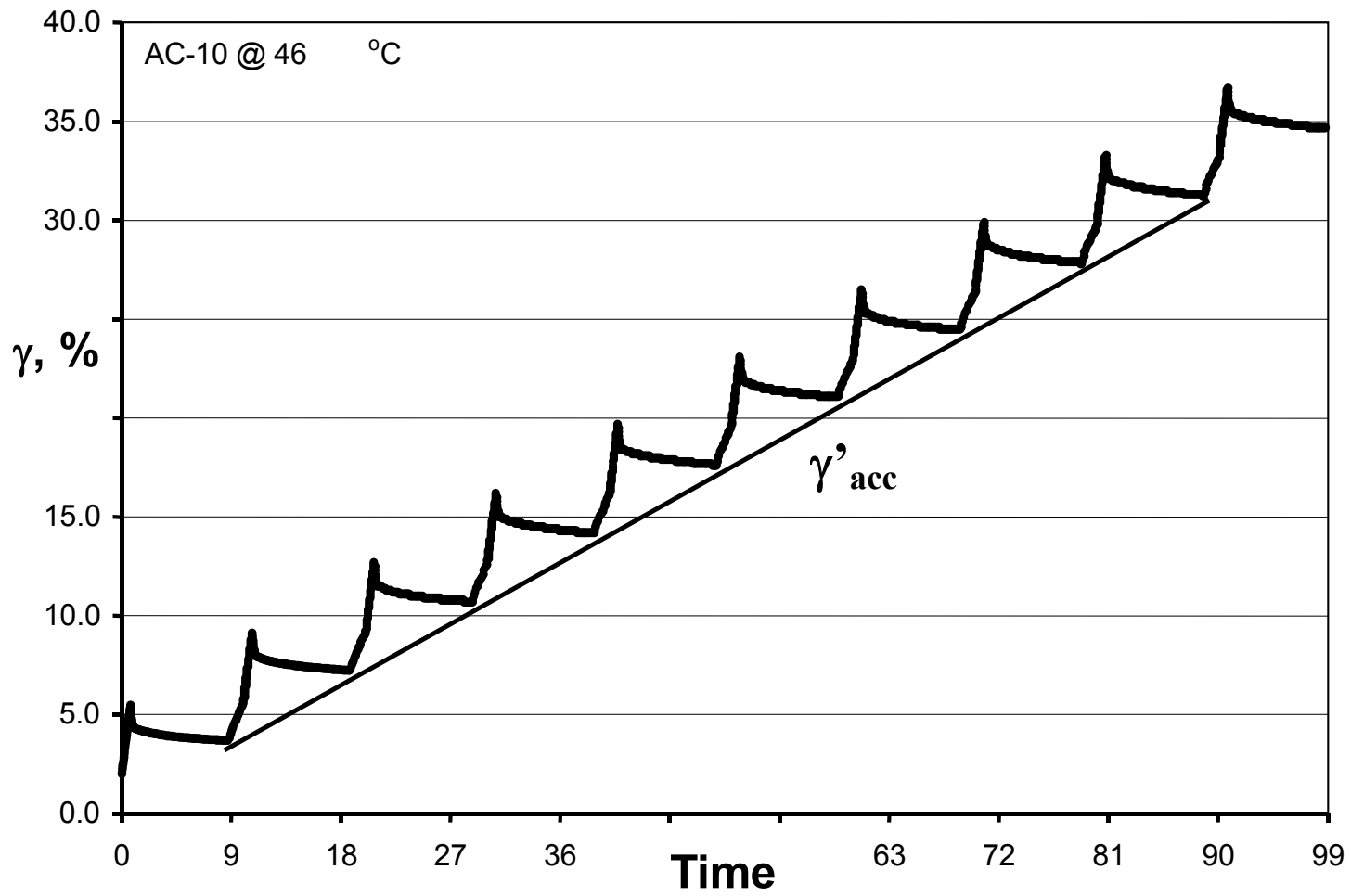
$$J(t) = (n-1)A + 1/\eta_0 c + \sum_{i=1}^{N-1} J_i \left(-e^{-\frac{t-10(n-1)-1}{c\Lambda_i}} - e^{-\frac{t-10(n-1)}{c\Lambda_i}} \right)$$

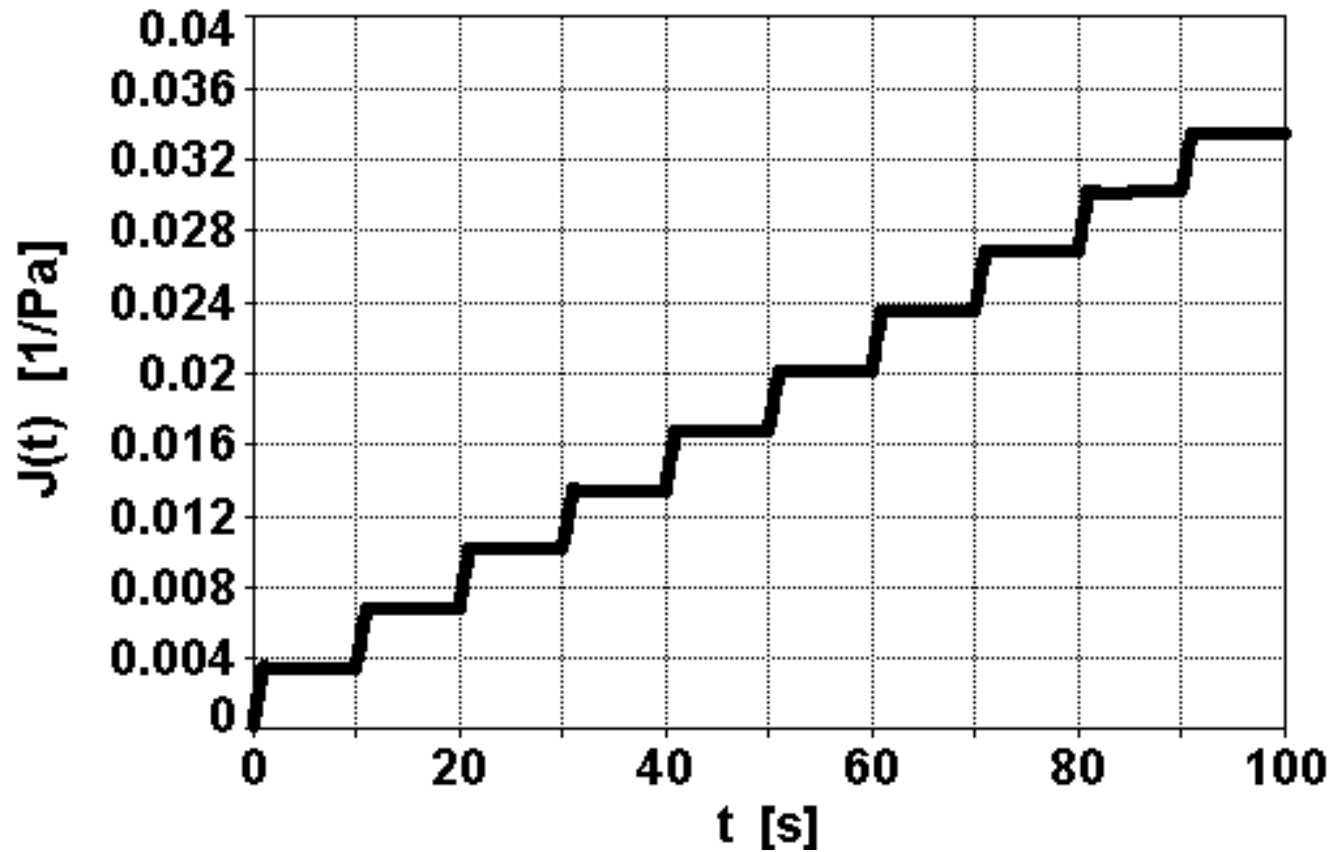
where

$$A = 1/\eta_0 c + \sum_{i=1}^{N-1} J_i \left(e^{-\frac{9}{c\Lambda_i}} - e^{-\frac{10}{c\Lambda_i}} \right)$$

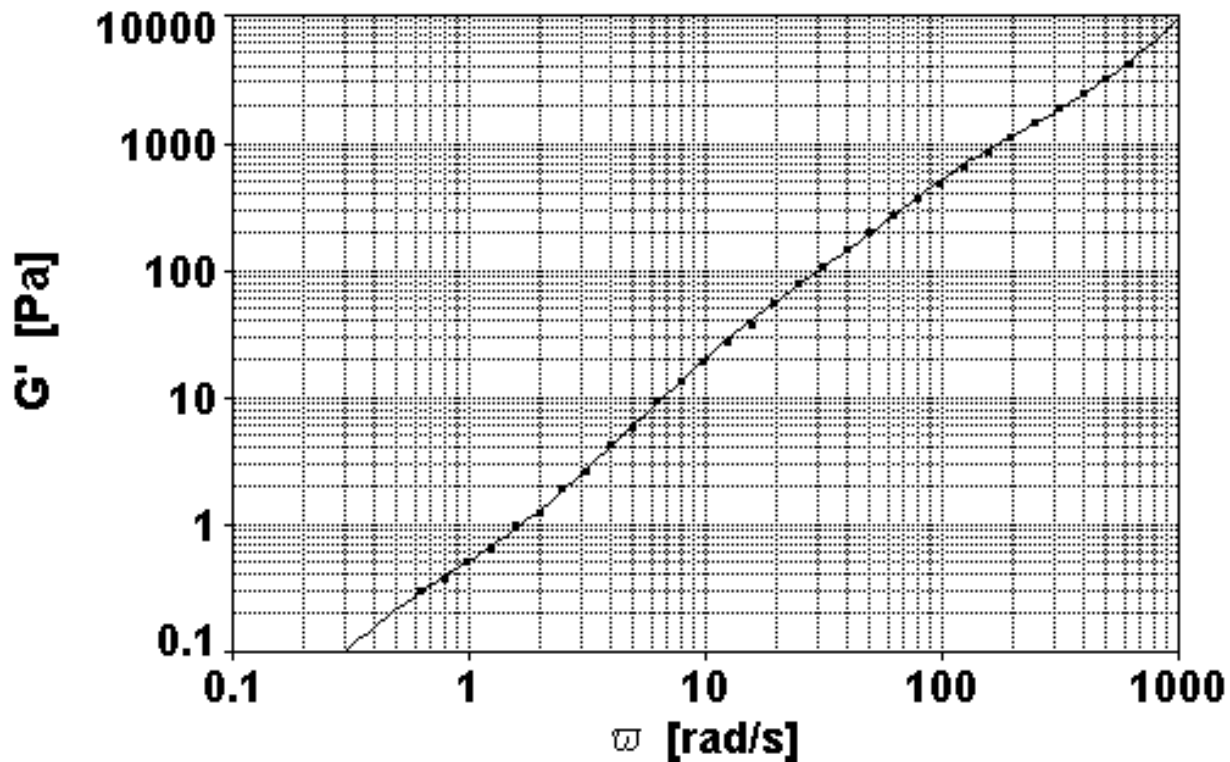
TABLE 1. ASPHALT BINDERS EVALUATED IN THIS STUDY

Asphalt Binder	Performance History Available
PG64-22- Conventional	Nevada D.O.T
AC20P-PMA	Nevada D.O.T
Cariphalte DM-PMA	Anecdotal in Europe





**Creep compliance $J(t)$, $T = 64^\circ \text{C}$.
First ten cycles. Nevada 64-22**

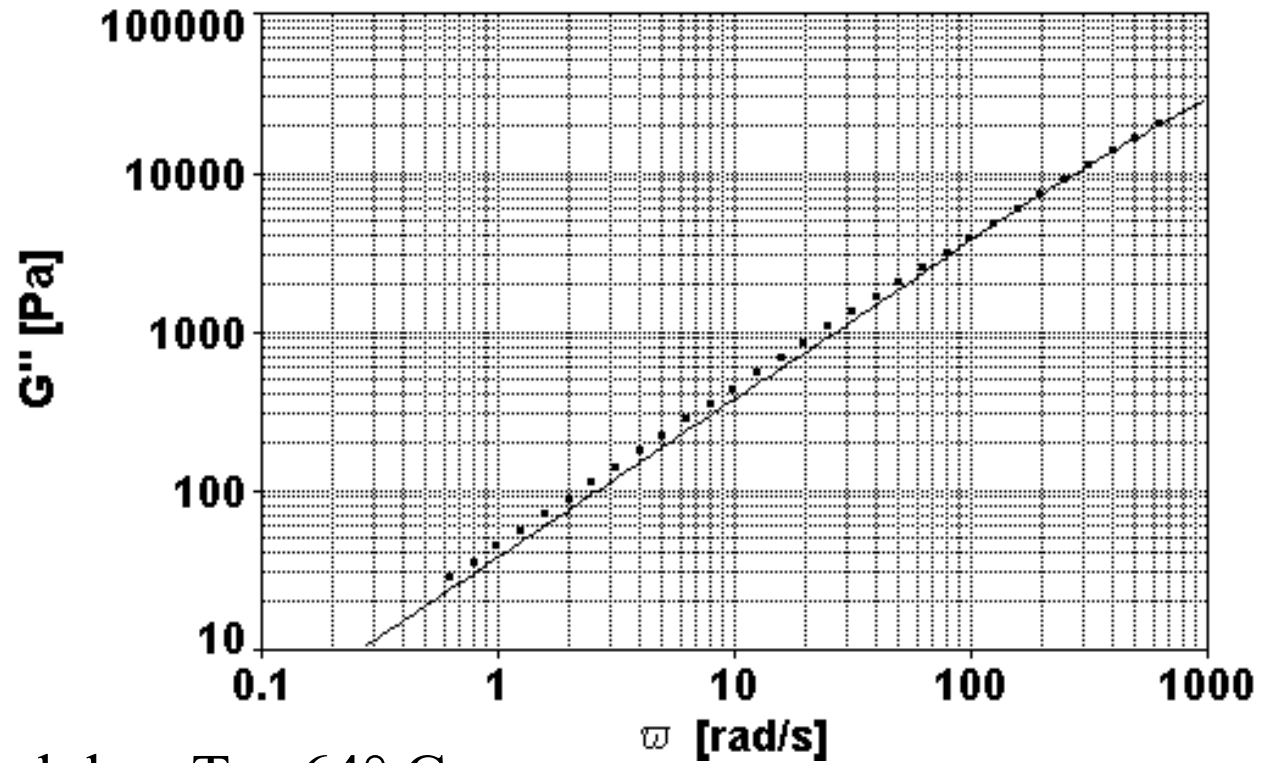


Storage modulus, $T = 64^\circ \text{C}$.

■ experimental,

— calculated from the discrete relaxation spectrum (4 modes).

Nevada 64-22.

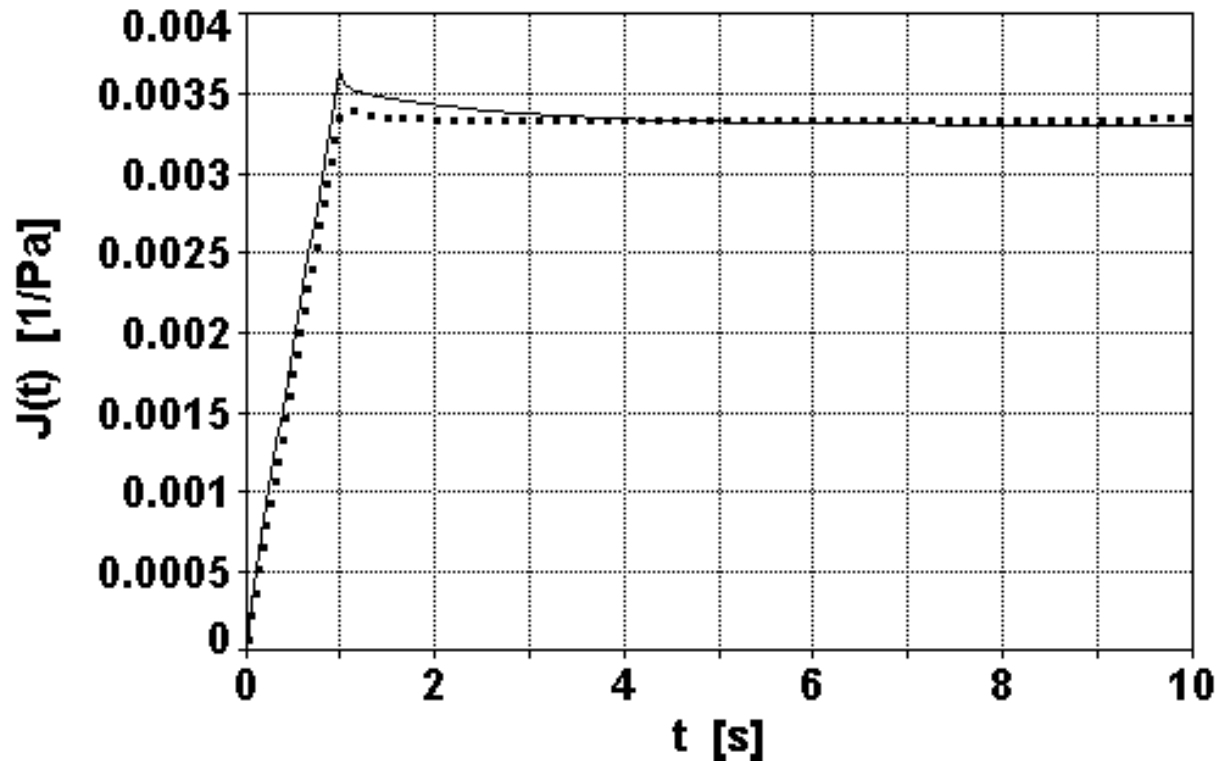


Loss modulus, $T = 64^\circ \text{C}$.

■ experimental,

— calculated from the discrete relaxation spectrum (4 modes).

Nevada 64-22.

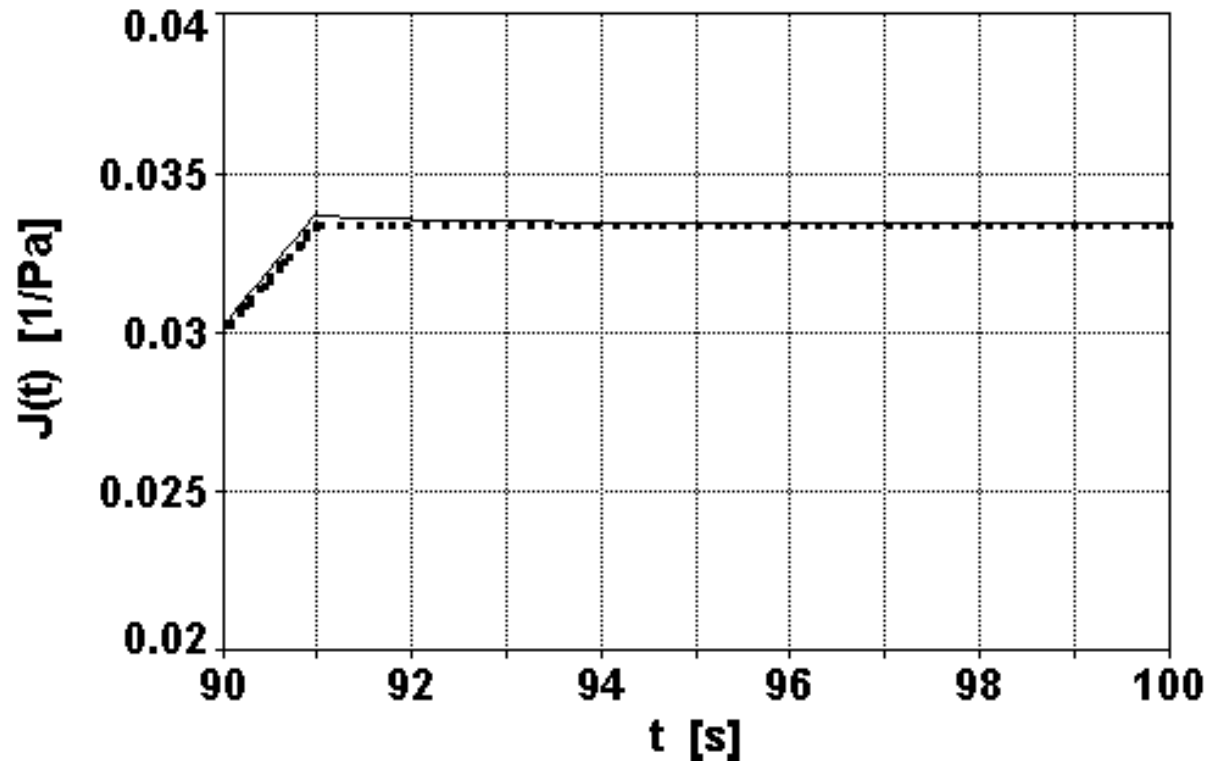


First cycle, $T = 64^{\circ} \text{C}$.

■ experimental,

▨ calculated from Eqs. (13)-15).

Nevada 64-22.

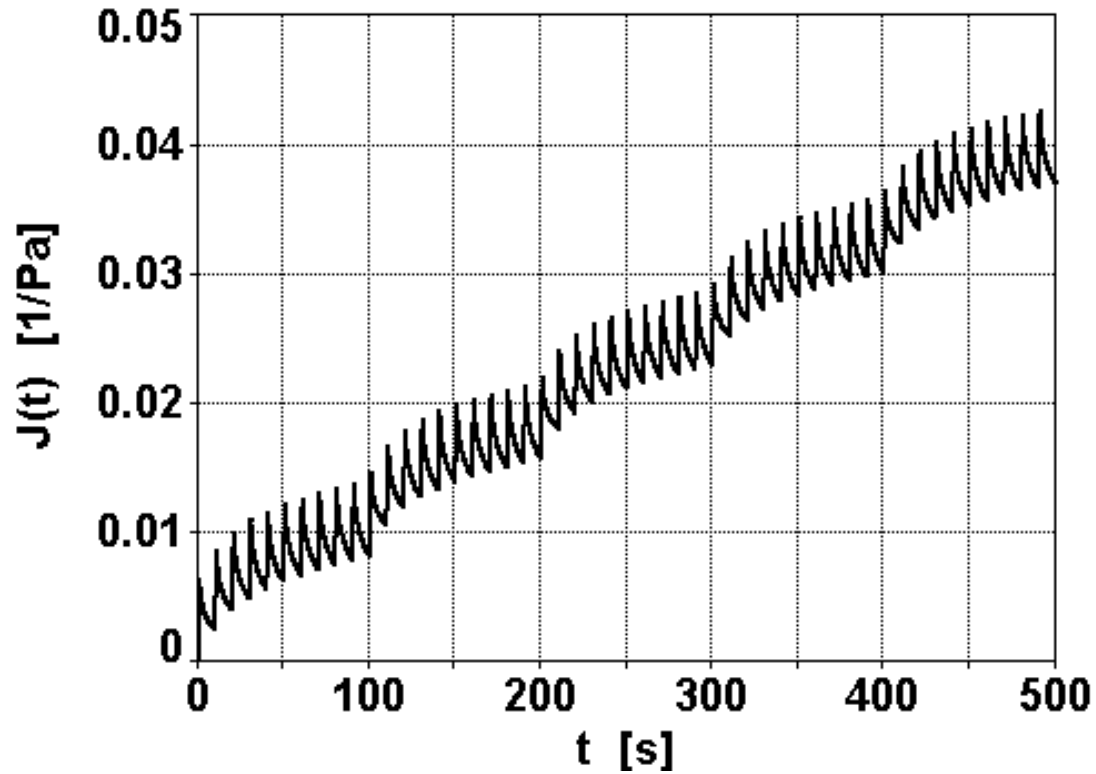


Cycle n=10, T = 64° C.

■ experimental,

▨ calculated from Eqs. (13)-(15).

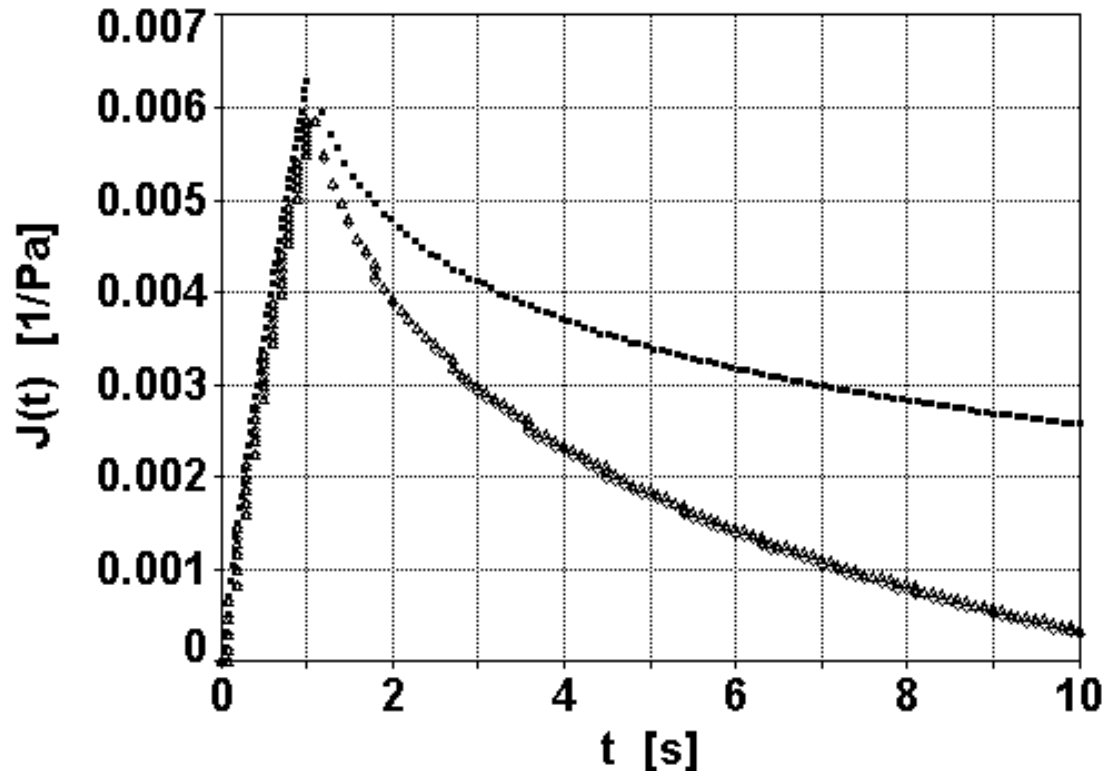
Nevada 64-22.



Creep compliance $J(t)$,

$T = 64^\circ \text{C}$. First fifty cycles.

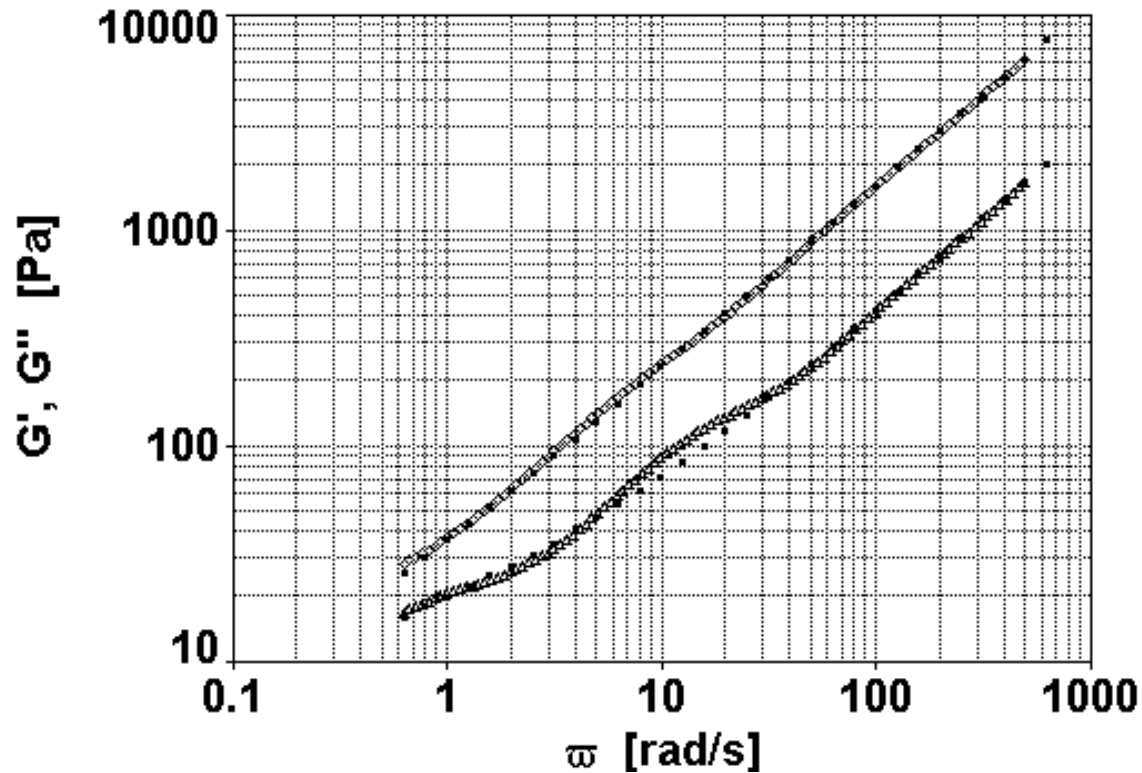
Nevada AC20P.



Creep compliance $J(t)$, $T = 64^\circ\text{C}$.

cycle 1, Δ cycle 10, \diamond cycle 50.

Nevada AC20P.

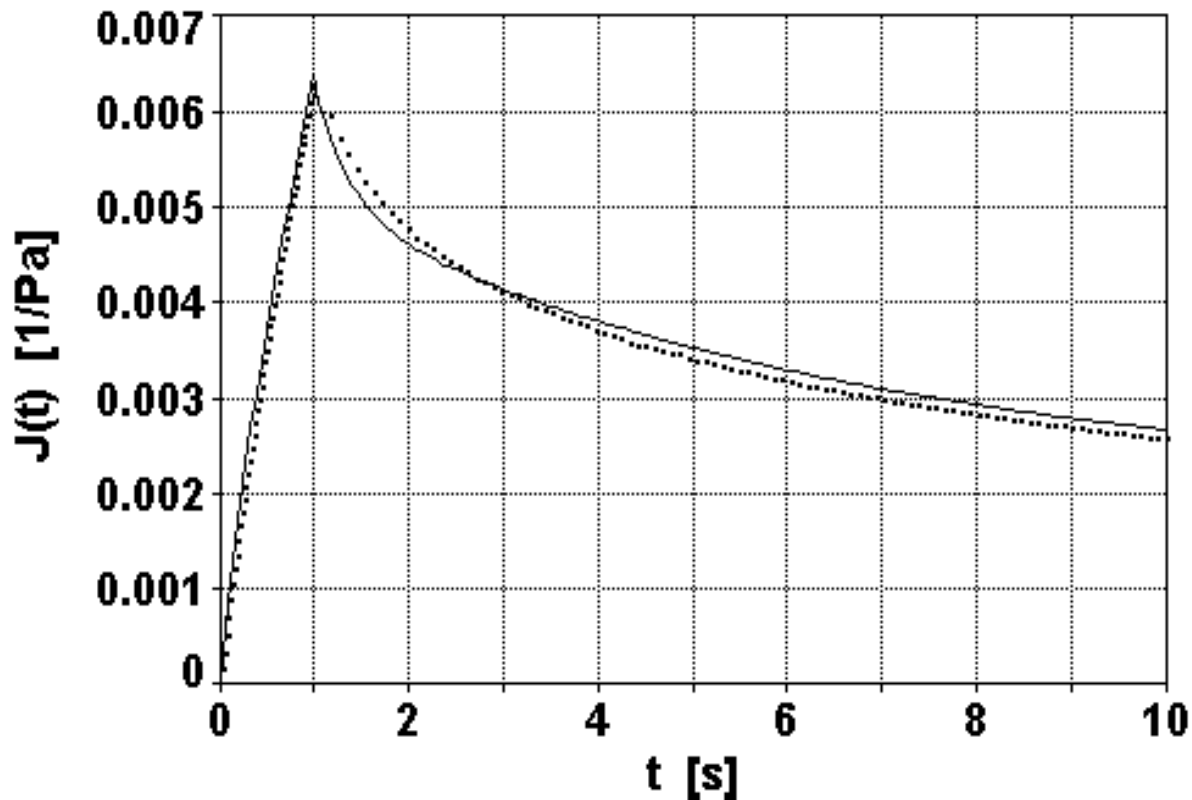


Dynamic moduli, $T = 64^\circ\text{C}$.

Experimental: $\Delta G'$, $\diamond G''$,

■ calculated from the discrete relaxation spectrum (5 modes).

Nevada AC 20P.



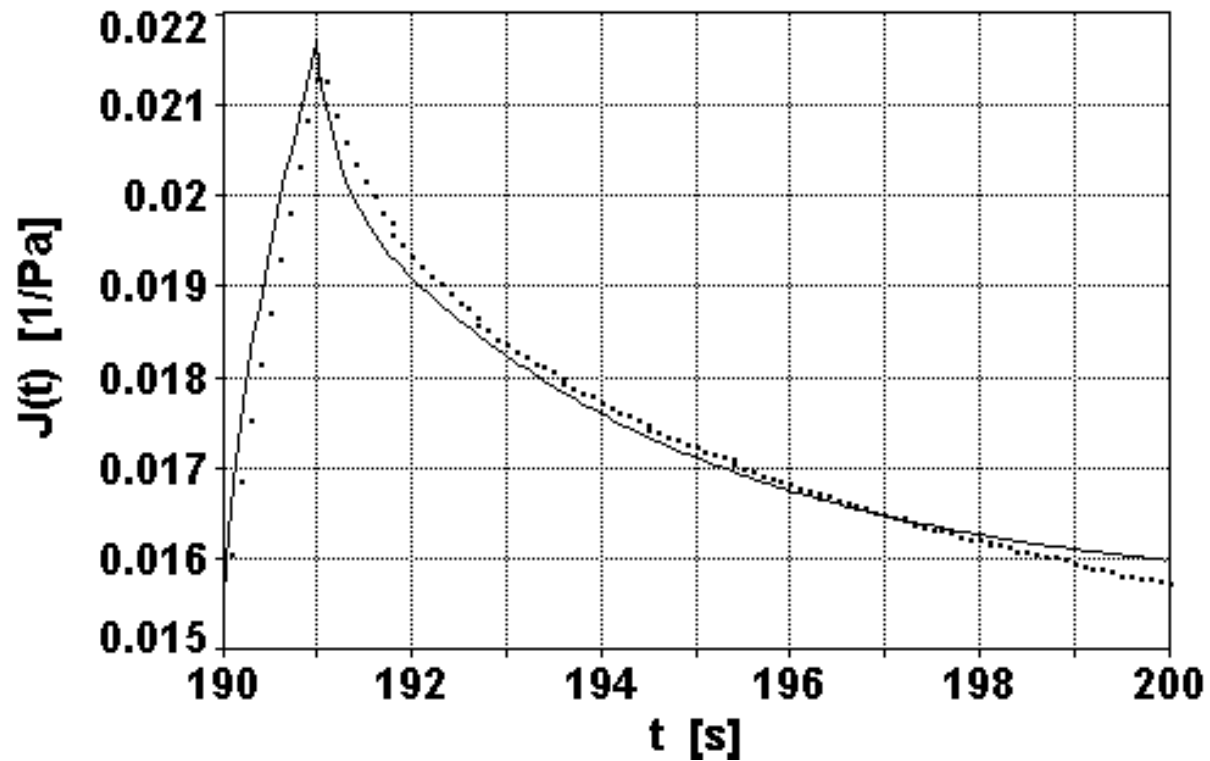
First cycle - repeated creep and recovery test, $T = 64^{\circ} \text{C}$.

■ experimental,



calculated from Equs. (13)-(15).

Nevada AC20P.

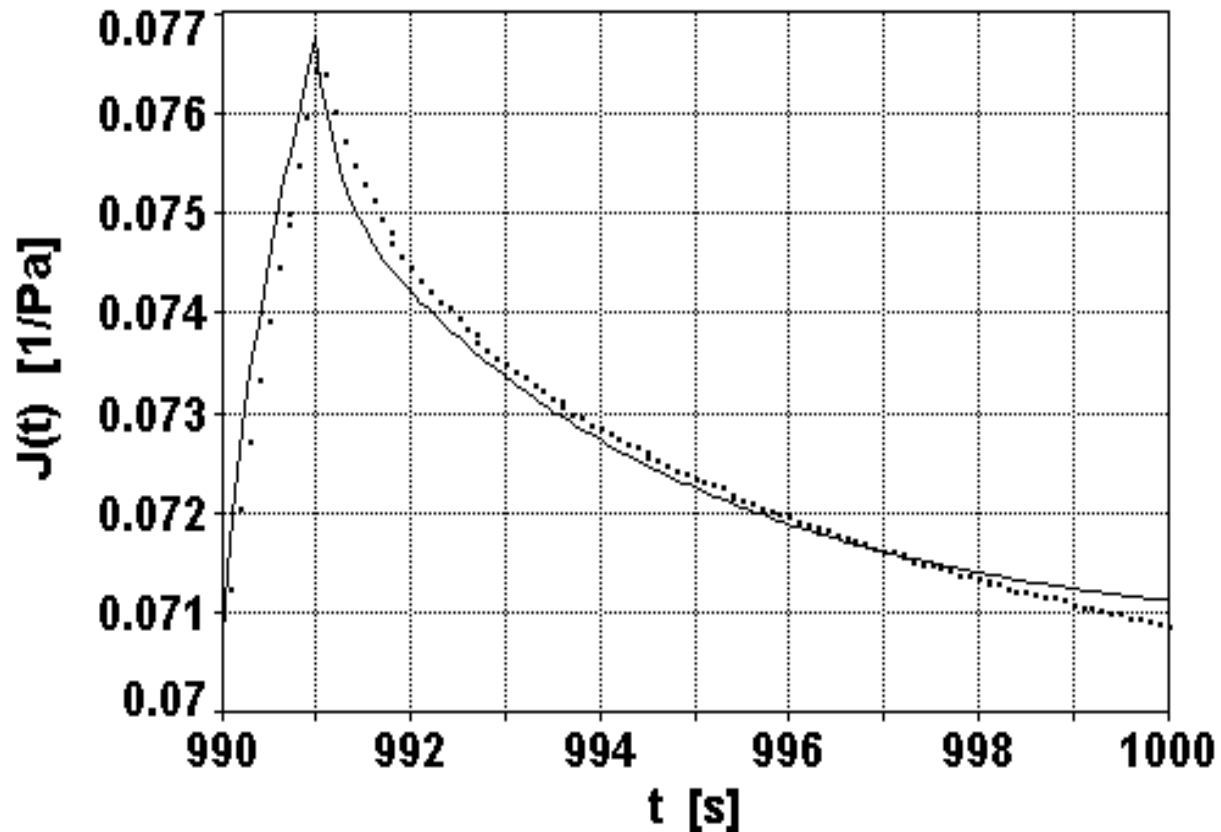


Cycle, $n = 50$, $T = 64^\circ \text{C}$.

■ experimental,

⊠ calculated from Eqs. (13)-(15).

Nevada AC20P.

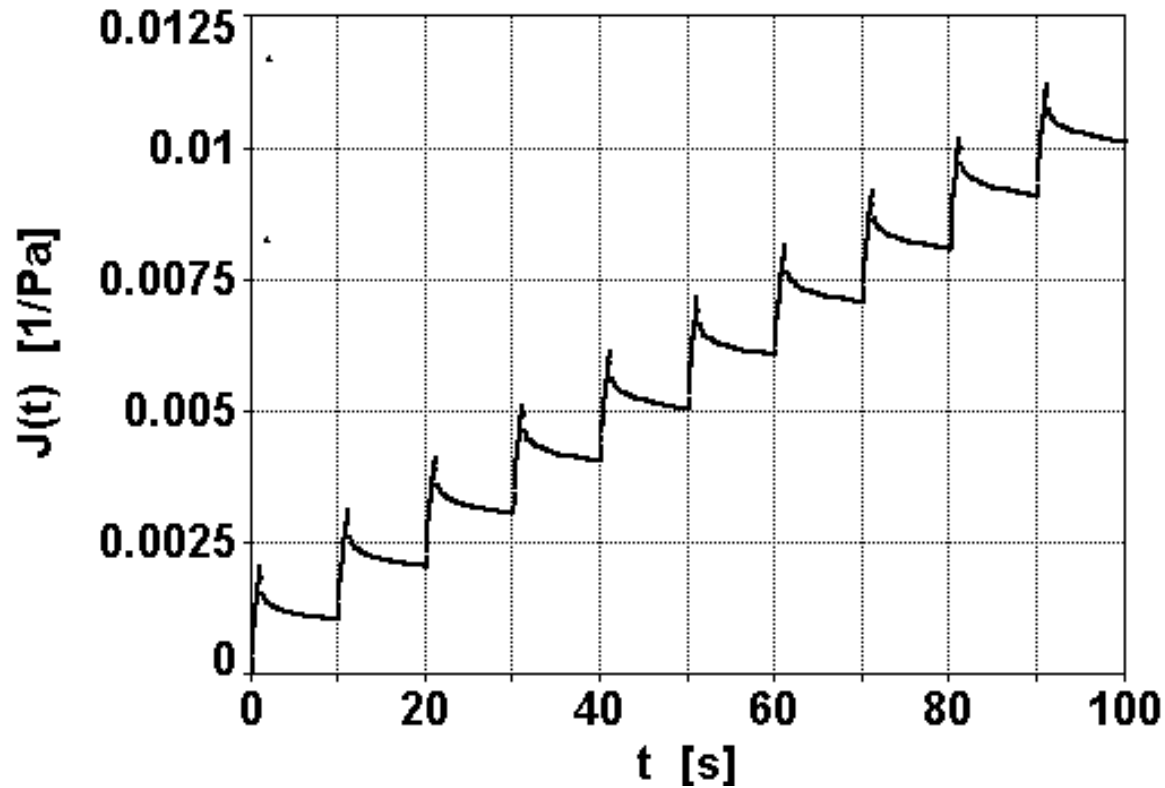


Cycle, $n = 100$, $T = 64^\circ \text{C}$.

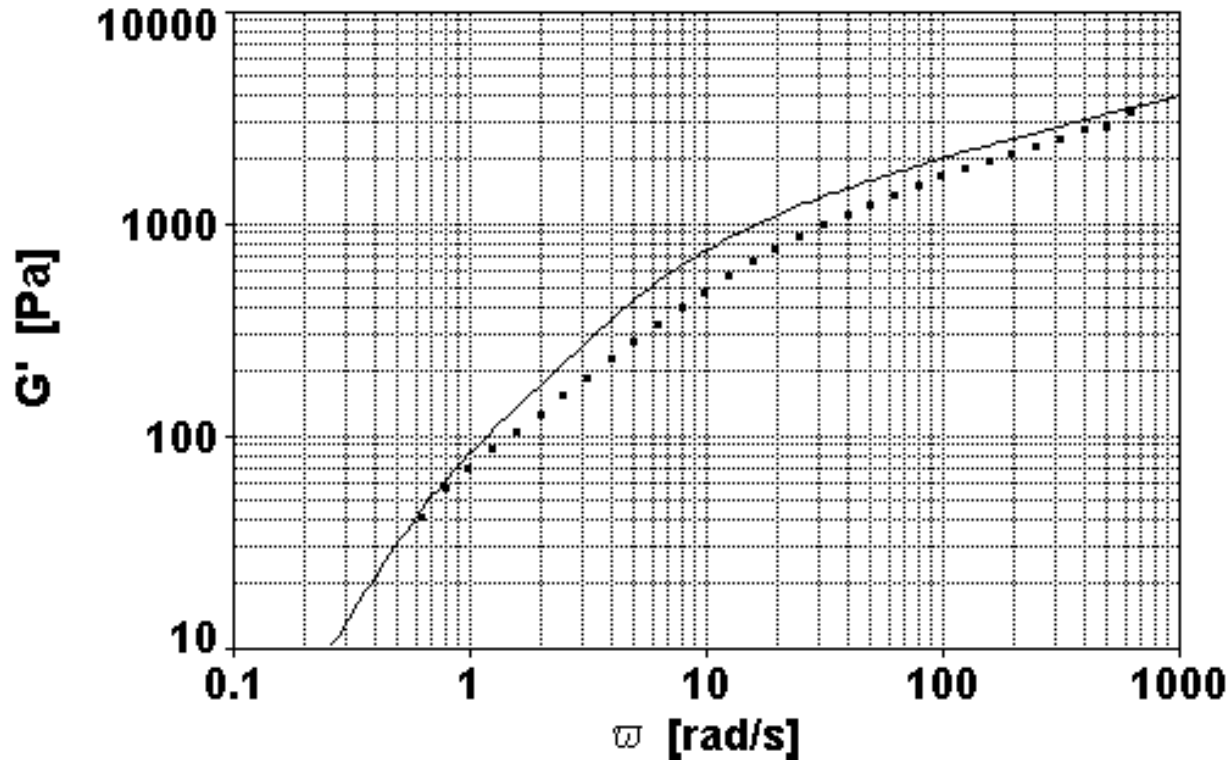
■ experimental,

▣ calculated from Eqs. (13)-(15).

Nevada AC20P.



**Creep compliance $J(t)$, $T = 76^\circ \text{C}$. First ten cycles.
Cariphalt 76.**

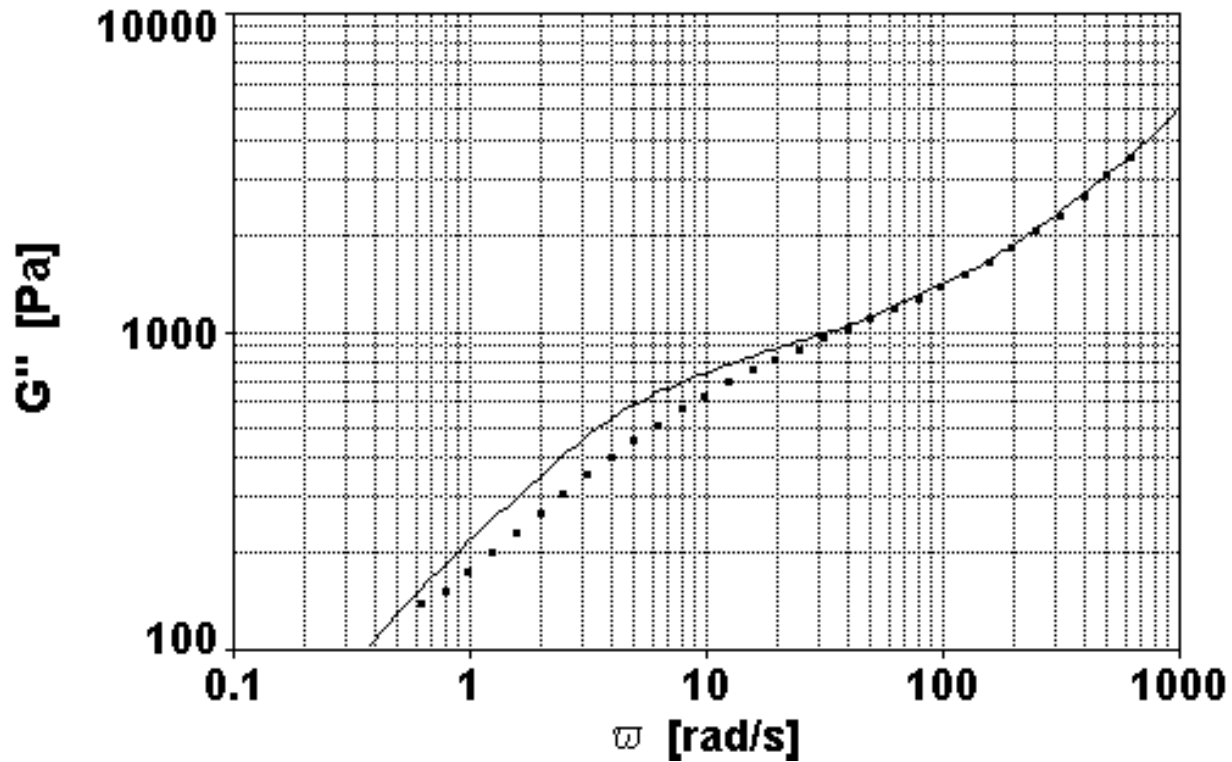


Storage modulus, $T = 76^\circ \text{C}$.

■ experimental,

— calculated from the discrete relaxation spectrum (6 modes).

Cariphalt 76.

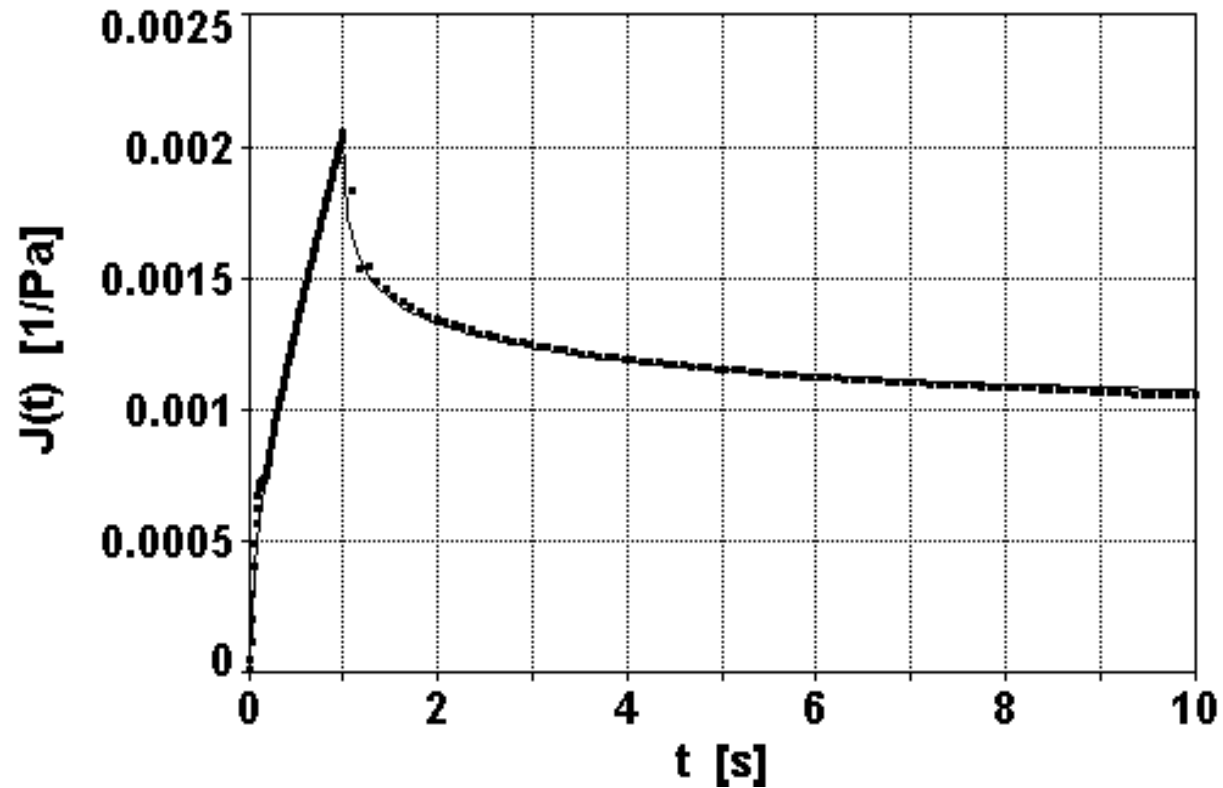


Loss modulus, $T = 76^\circ \text{C}$.

■ experimental,

— calculated from the discrete relaxation spectrum (6 modes).

Cariphalt 76.

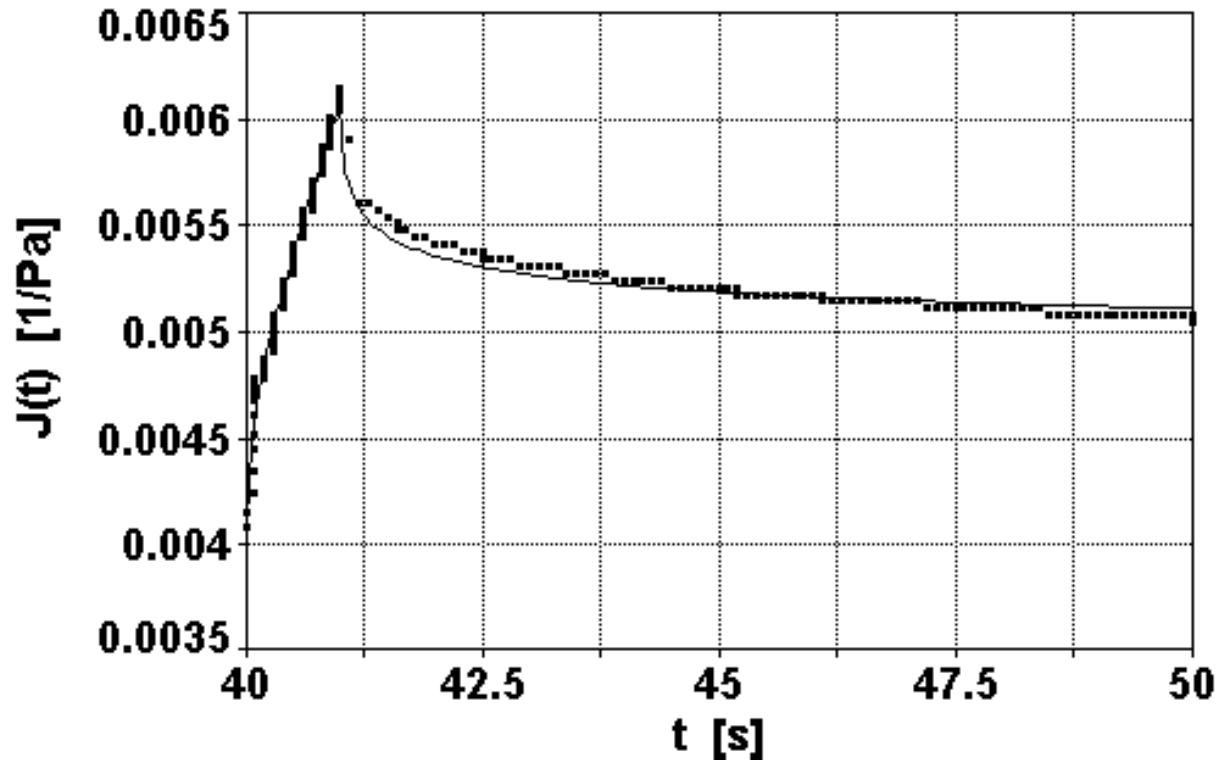


Creep compliance $J(t)$, $T = 76^\circ \text{C}$. First cycle,

■ experimental,

▬ calculated from Eqs. (13)-(15).

Cariphalt 76.

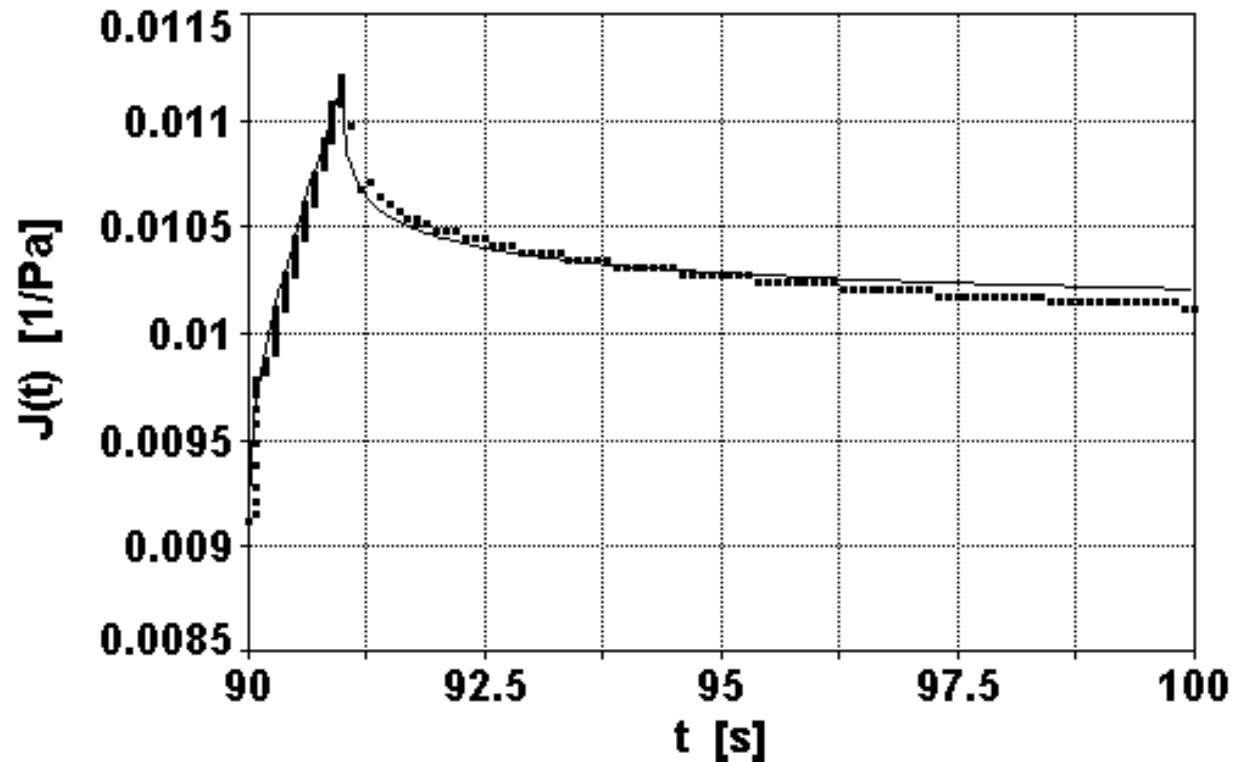


Creep compliance $J(t)$, $T = 76^\circ \text{C}$. Fifth cycle,

■ experimental,

▬ calculated from Eqs.(13)-(15).

Cariphalt 76.



Creep compliance $J(t)$, $T = 76^\circ \text{C}$. Cycle $n = 10$,

■ experimental,

▨ calculated from Eqs. (13)-(15).

Cariphalt 76.

CONCLUSION

The Boltzmann superposition principle governs the repeated creep and recovery test in the linear viscoelastic domain of small deformations. For the test, set to perform creep during the first one second followed by nine seconds of recovery, and repeated n-times one has:

$$\text{Creep } J(t) = J(t-10(n-1)) + (n-1)(J(10)-J(9))$$

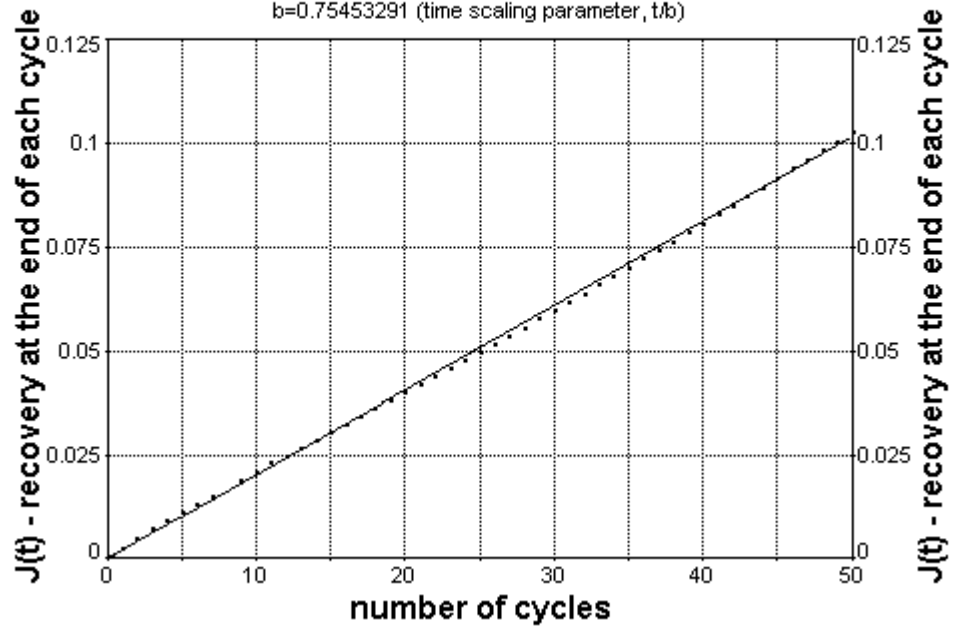
$$\text{Recovery } J(t) = J(t-10(n-1))-J(t-10(n-1)-1) + (n-1)(J(10)-J(9))$$

sample "a" NevadaAC20PCR64C

$r^2=0.99923749$ DF Adj $r^2=0.99920504$ FitStdErr=0.00081788316 Fstat=62901.942

a=372.93722 (zero shear viscosity)

b=0.75453291 (time scaling parameter, t/b)

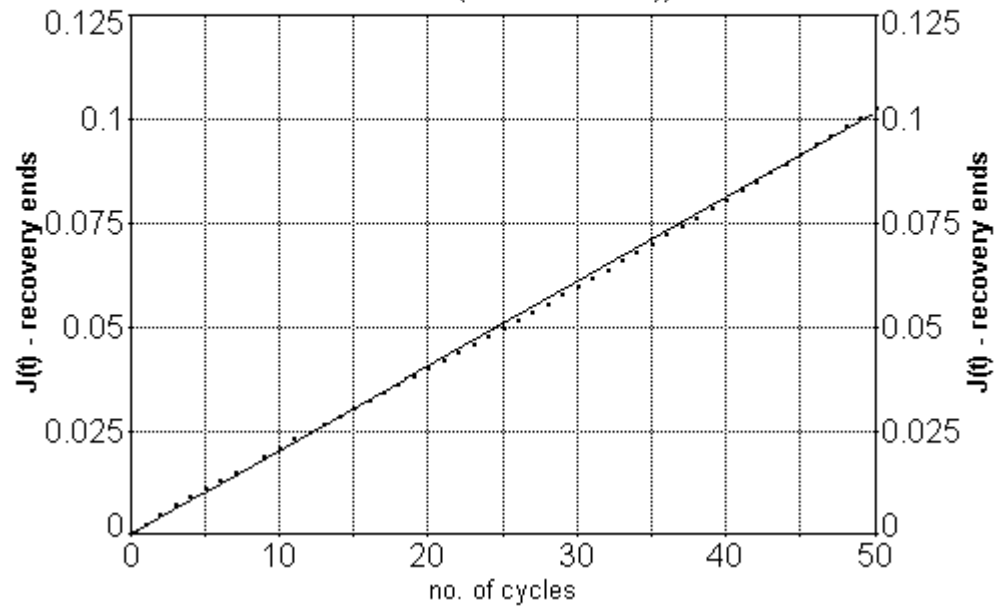


sample "a" NevadaAC20PCR64C

only viscosity term

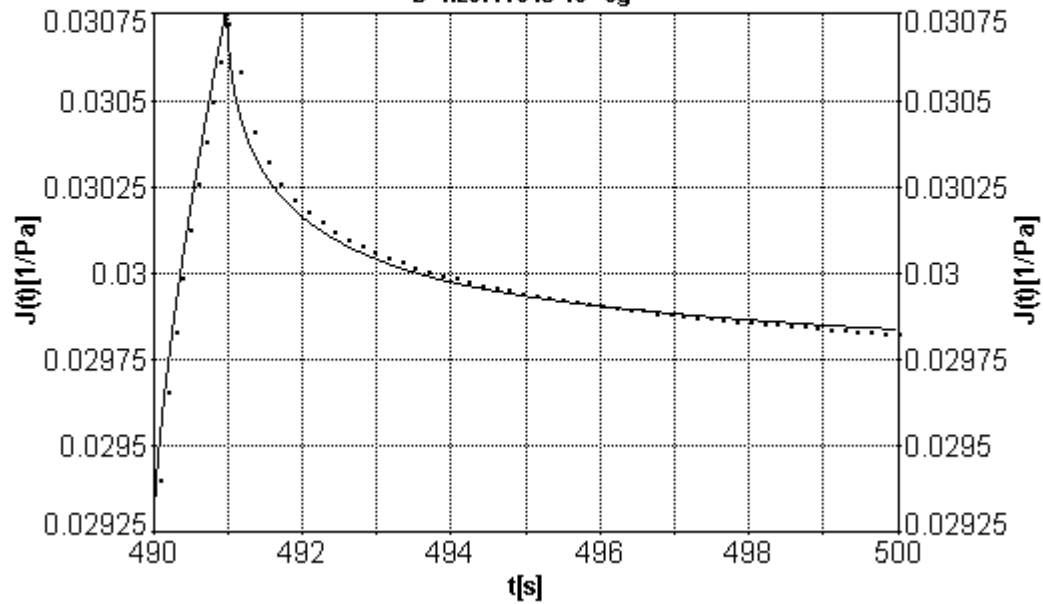
$r^2=0.99923749$ DF Adj $r^2=0.9992216$ FitStdErr=0.00080949439 Fstat=64212.399

a=492.39291 (zero shear viscosity)



70C? 71.36dynamic
time scaling factor set to 1

$r^2=0.97958895$ DF Adj $r^2=0.97885998$ FitStdErr=3.3676596e-05 Fstat=2735.605
a=1690.8888 (1725 from spectra, however spectra from master @ 71.36C)
b=1.2877704e-13= Jg



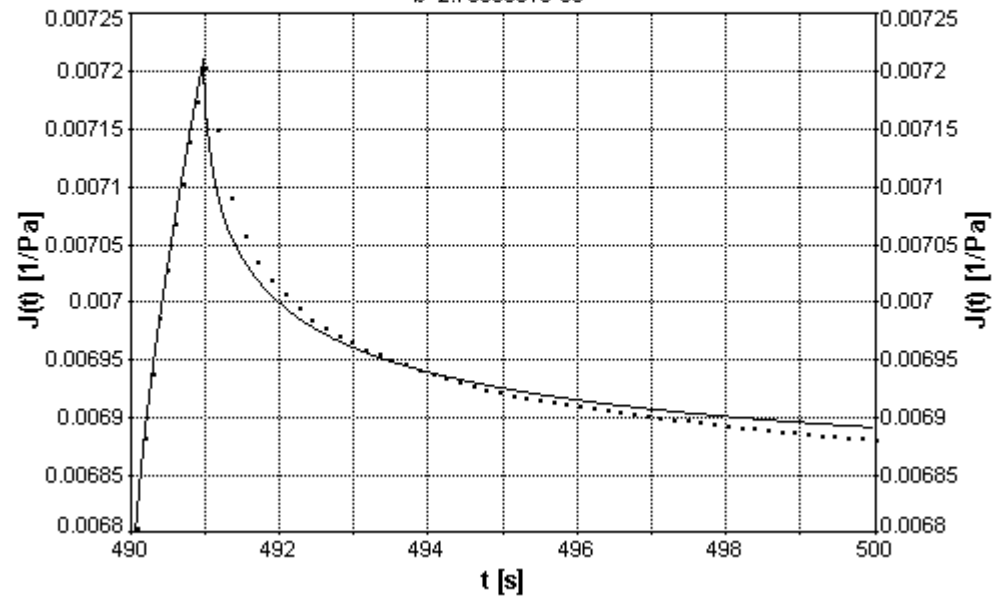
cycle50 T=58C,dynamic data@58.94C

time scaling factor set to 1

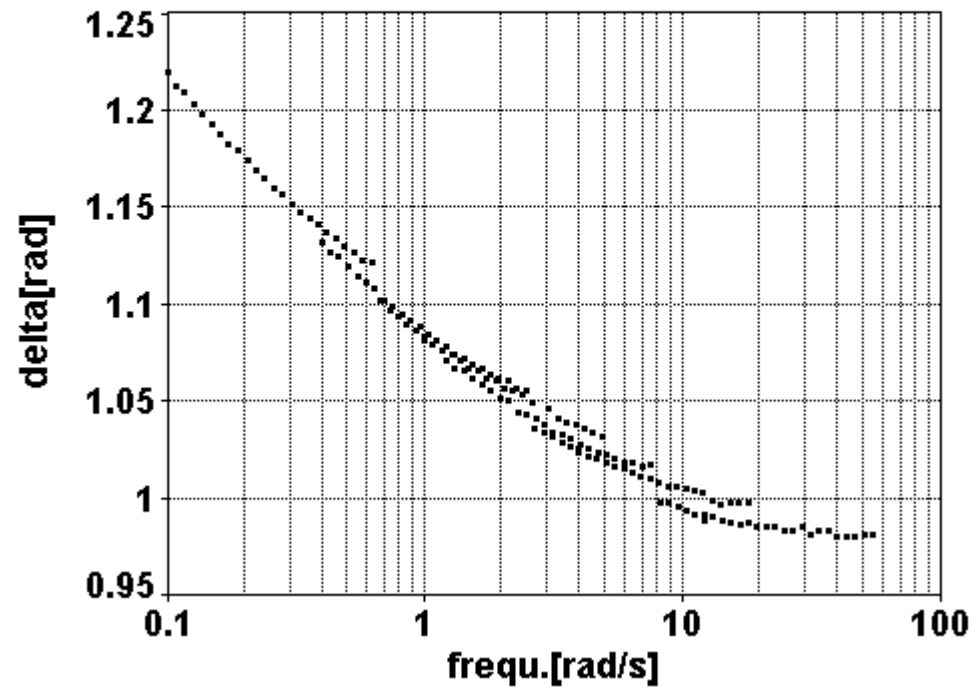
$r^2=0.9812622$ DF Adj $r^2=0.980593$ FitStdErr= $1.1336416e-05$ Fstat= 2984.98

a=7498.2867 (1.590e+04 from spectra @ 58.94C)

b=2.7856061e-05



Phase angle, 3 temps. (58C, 65C, 71C)
shifted to Tr=71C



70C(Reinke) only 70C for spectra time scaling 1
from spectra - a -(zero-shear viscosity)=1741

$r^2=0.98327334$ DF Adj $r^2=0.98269656$ FitStdErr=3.2828628e-05 Fstat=3468.3027
a=1704.9998
b=1.4147326e-13

