NONLINEAR COMPLEX MODULUS IN BITUMENS

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Large Amplitude Oscillations - LAOS

Input: shear strain - sinusoidal simple shear ($\gamma_0 >> 1$)

$$\gamma(t) = \gamma_0 \sin(\omega t)$$

Output: shear stress - periodic (FFT or DFT)

$$\tau(t) = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \tau_n(\omega, \gamma_0) \sin\left(n\omega t + \delta_n(\omega, \gamma_0)\right)$$

Higher harmonic moduli:

$$G'_n = (\tau_n / \gamma_0)\cos\delta_n, \; G''_n = (\tau_n / \gamma_0)\sin\delta_n$$

$$\tau(t) = \gamma_0 \sum_{\substack{n=1 \\ \text{odd}}} \left[G'_n(\omega, \gamma_0)\sin(n\omega t) + G''_n(\omega, \gamma_0)\cos(n\omega t) \right]$$

Linear viscoelastic limit, $\gamma_0 \rightarrow 0$:

n = 1, and G'1 ≡ G'(γ), G''1 ≡ G''(γ)

Nonlinear viscoelastic constitutive equations
Many available - a few practical - none satisfactory in both shearing and elongational flows
Wagner's modification of Lodge's rubberlike- liquid :
memory function can be factorized

memory function = linear viscoelastic memory
x
damping function

In LAOS the damping function, h, is a double periodic function of the current time and the elapsed time (material has memory).

By developing, h, into a double Fourier series One obtains nonlinear harmonic moduli.

$$G'1(\omega, \gamma_0) = h_{0,0} G'_{lin} - \left|B_2^*\right| \cos \beta_2$$

$$G''1(\omega, \gamma_0) = h_{0,0} G''_{lin} - \left|B_2^*\right| \sin \beta_2$$

for $n \geq 3$**, odd**

$$G'_n(\omega, \gamma_0) = \left|C_{n-1}^*\right| \cos \gamma_{n-1} - \left|B_{n+1}^*\right| \cos \beta_{n+1}$$

$$G''_n(\omega, \gamma_0) = \left|C_{n-1}^*\right| \sin \gamma_{n-1} - \left|B_{n+1}^*\right| \sin \beta_{n+1}$$

Here, G'_{lin} **and** G''_{lin} **are the linear dynamic moduli, and** $B_n^* = \left|B_n^*\right| \exp(-i\beta_n)$, $C^* = \left|C_n^*\right| \exp(i\gamma_n)$ **are complex functions generated by the double Fourier expansion of the damping function, and** $h_{0,0}$ **is the averaged damping function over the domain** $(-\pi/\omega,\ \pi/\omega) x (-\pi/\omega,\ \pi/\omega)$**.**

Kazatchkov's hypothesis (observed in some polymer melts) :

$$G'1 = (\omega, \gamma_o) = G'_{lin}(\omega) h(\gamma_0)$$

then it should be possible to obtain G'1 (and G''1) from linear viscoelastic moduli and the damping function (usually $\exp(-\alpha\gamma_0)$ **or** $1/(1 + a\gamma_0^b)$ **) by simple shifting.**

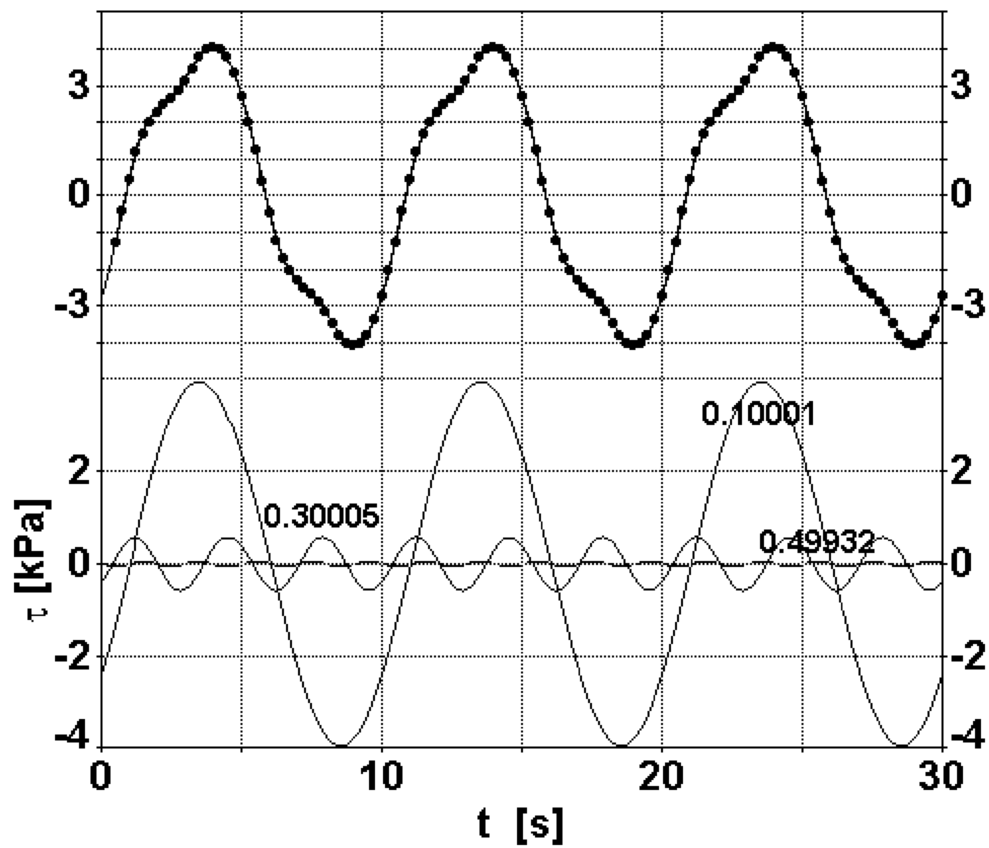
"Spoiling" terms

$$G'1(\omega, \gamma_0) = h_{0,0} G'_{lin} - \left| B_2^* \right| \cos \beta_2$$

$$G''1(\omega, \gamma_0) = h_{0,0} G''_{lin} - \left| B_2^* \right| \sin \beta_2$$

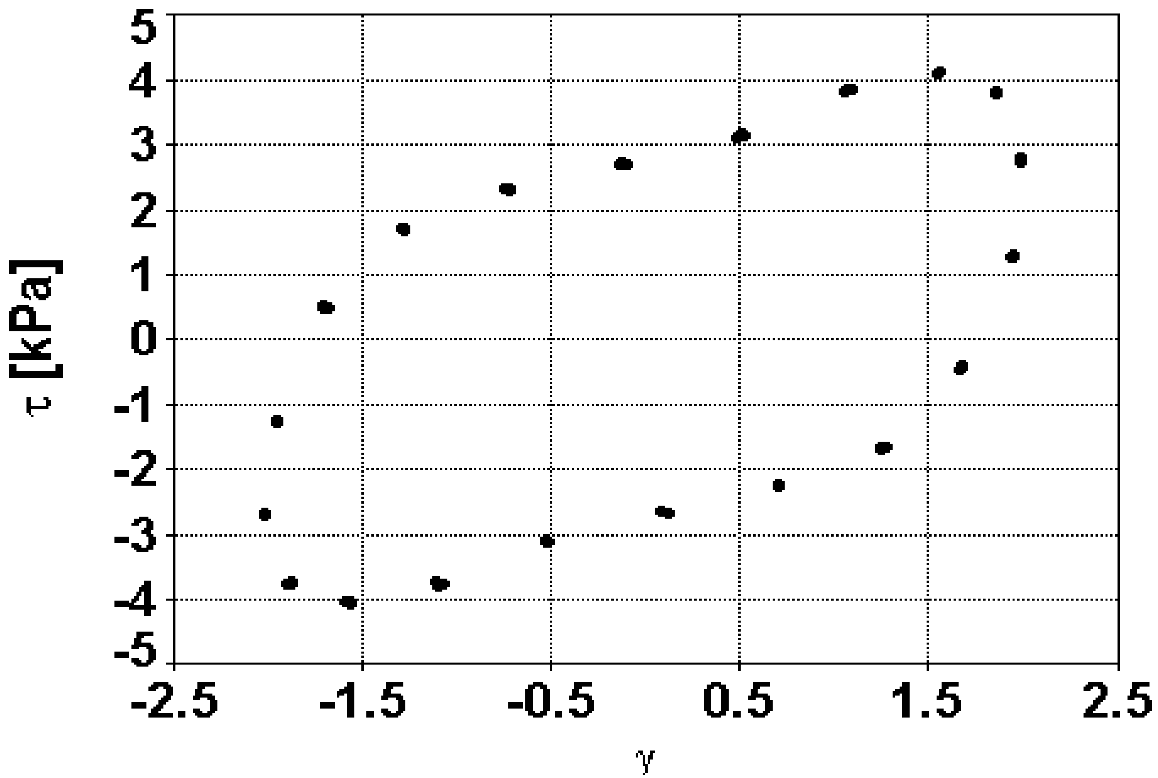
Fourier decomposition and reconstruction of the shear stress.

Frequency 1Hz, strain amplitude 4.

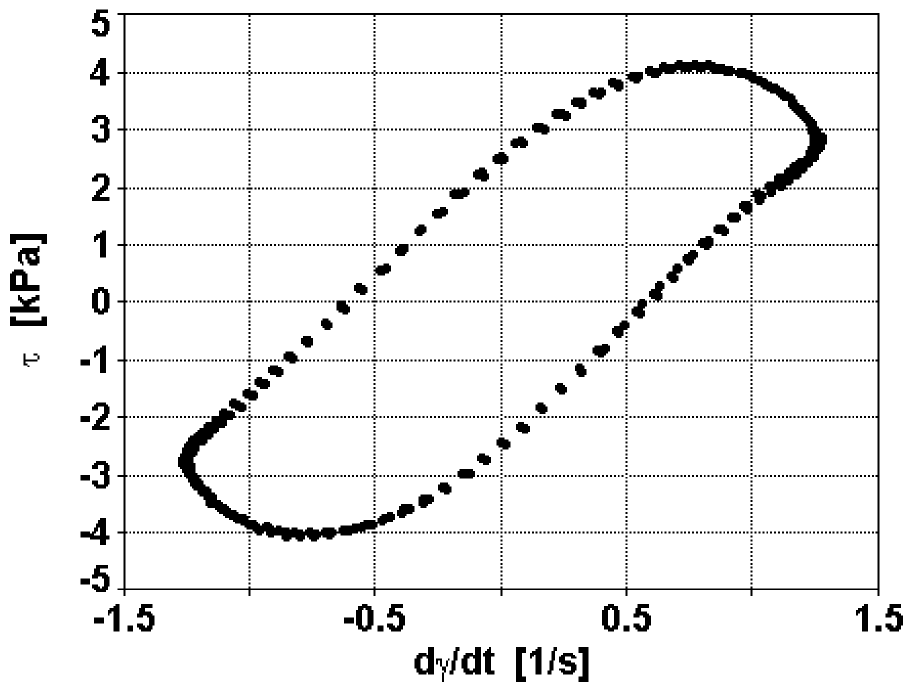


Lissajous figure

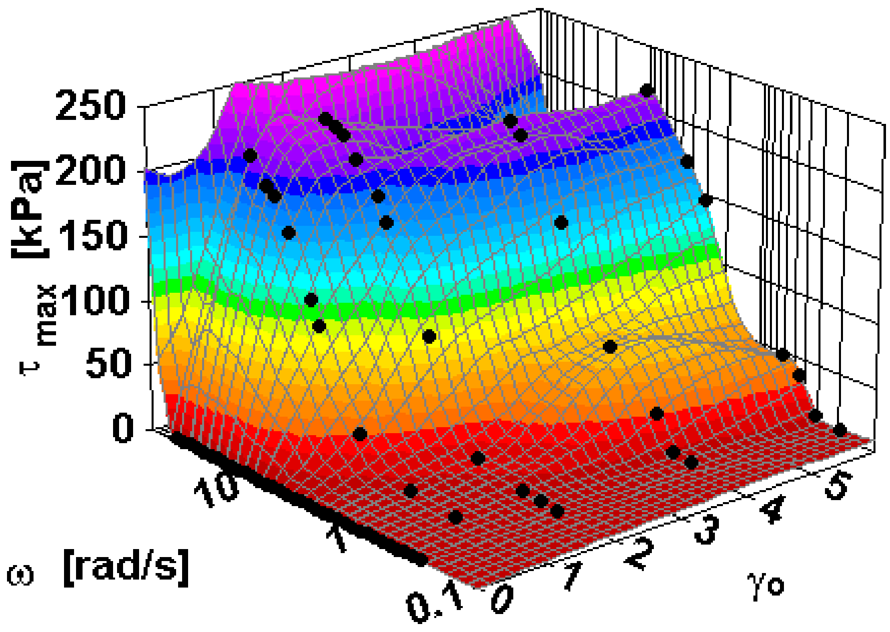
Frequency 1Hz, strain amplitude 4



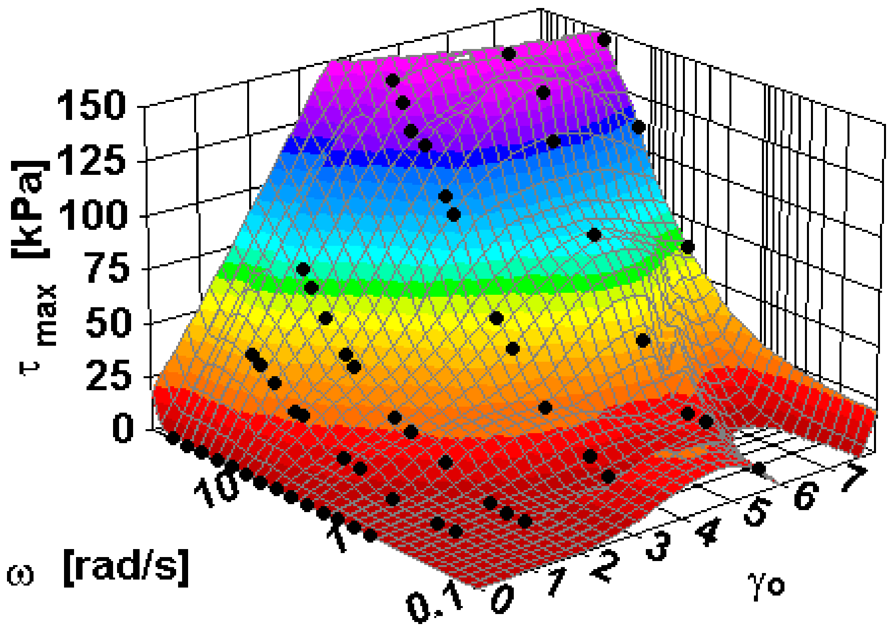
Same as in previous Fig. except the rate of strain is used, and some points are interpolated.



Maximum shear stress. Base asphalt, T = 27C.

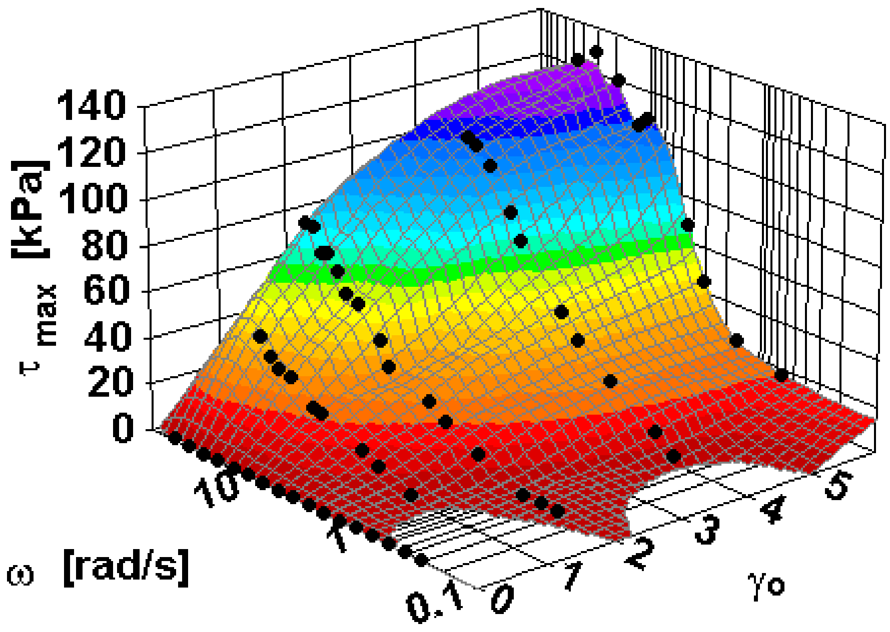


Maximum shear stress. Base asphalt with 4% SBS, T = 44C



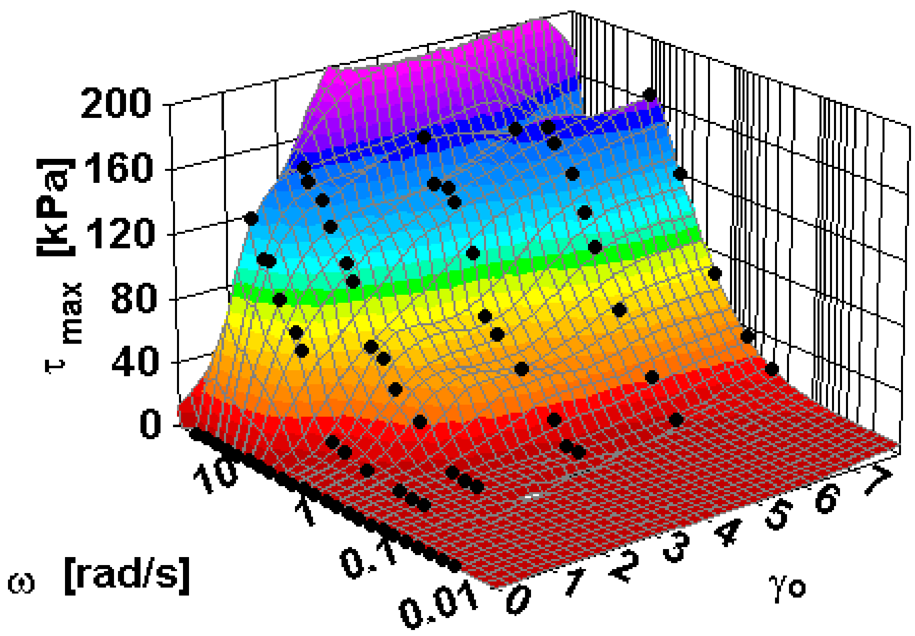
Maximum shear stress. Base asphalt with 6% SBS,

T = 50C.



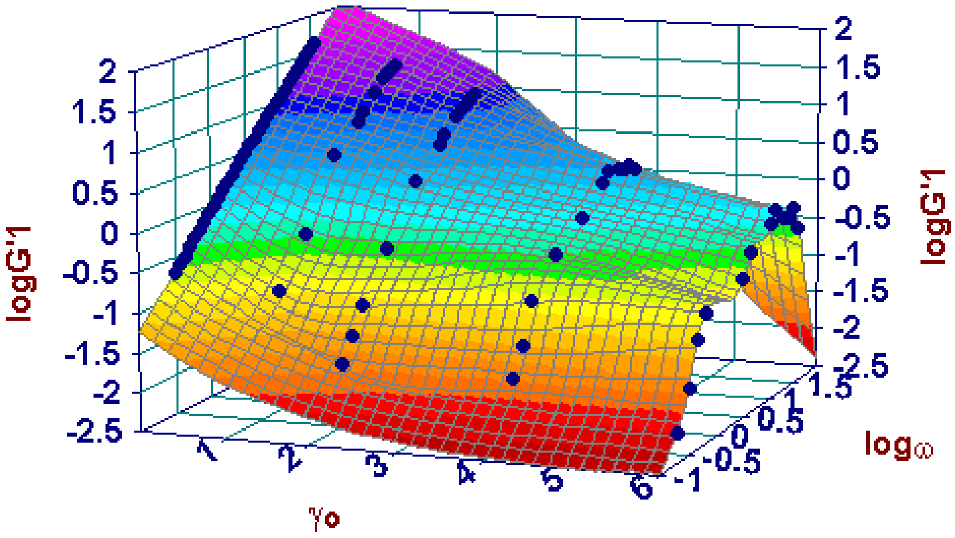
Maximum shear stress. Base asphalt with 4% EVA,

T = 34C.

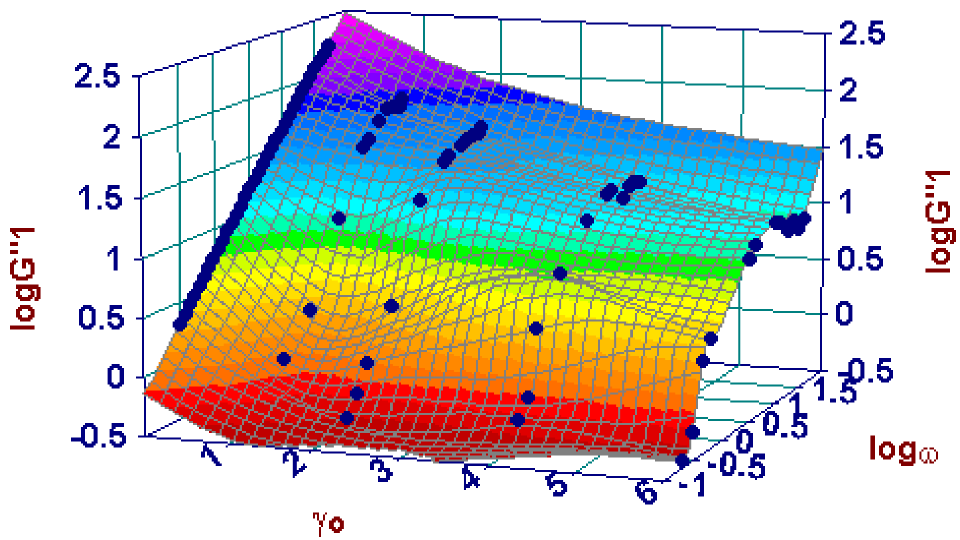


First harmonic modulus G'1. Base asphalt,

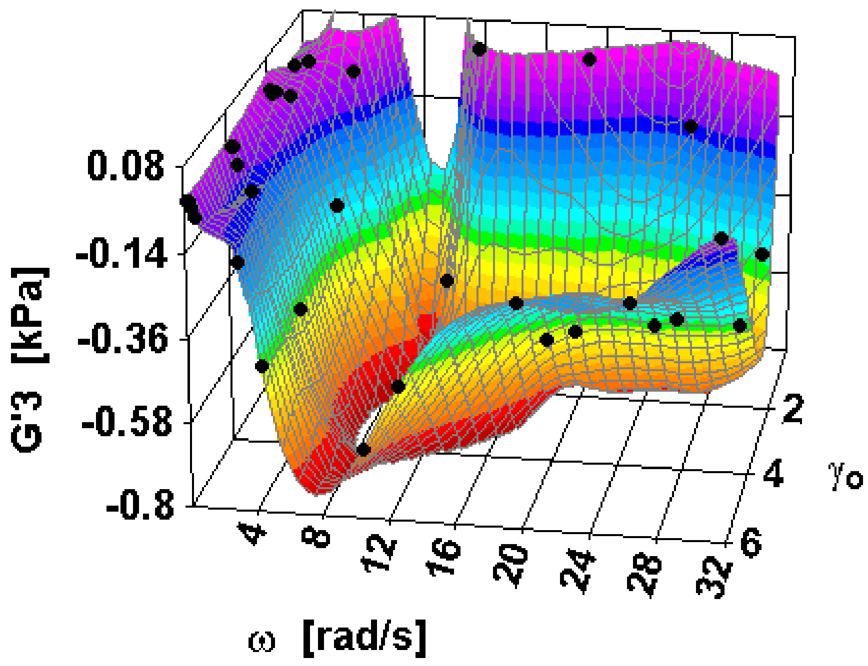
T = 27C.



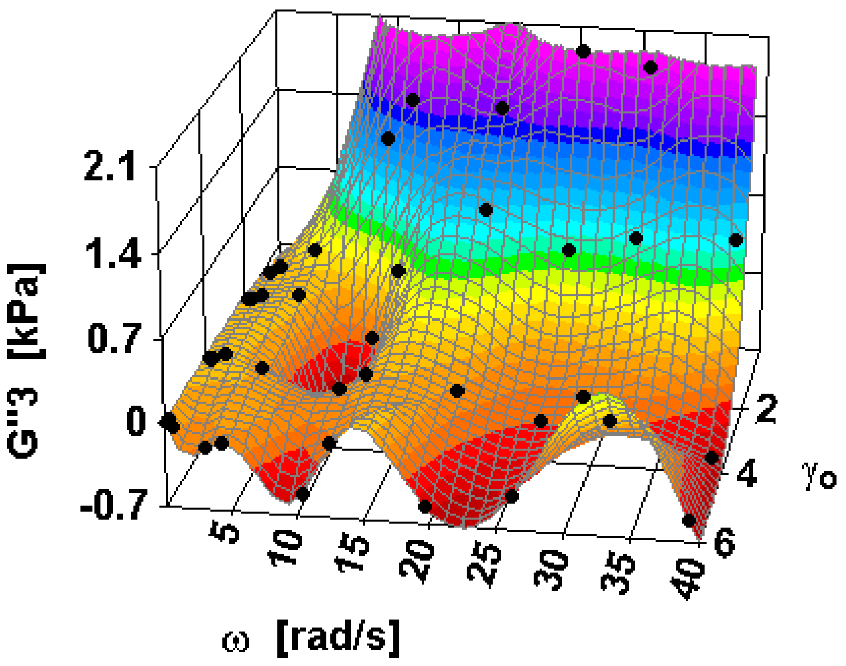
First harmonic modulus G''1. Base asphalt, T = 27C



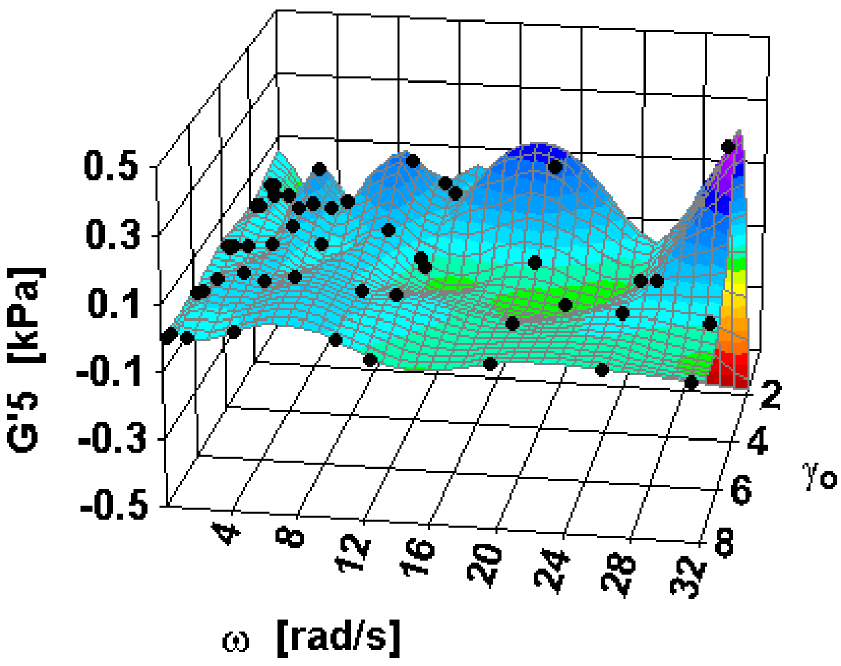
Third harmonic modulus G'3. Base asphalt, T = 27C.



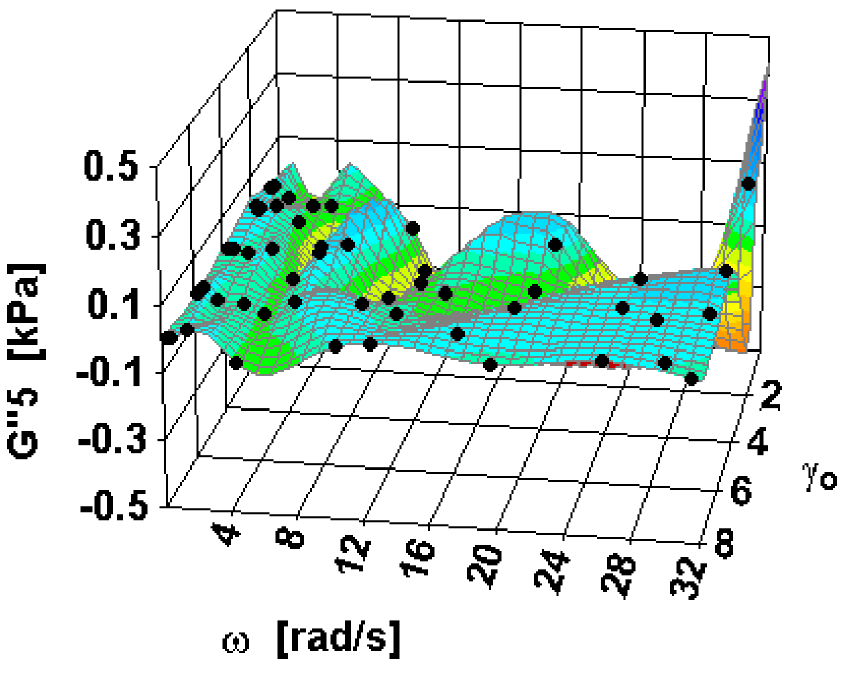
Third harmonic modulus G''3. Base asphalt, T = 27C.



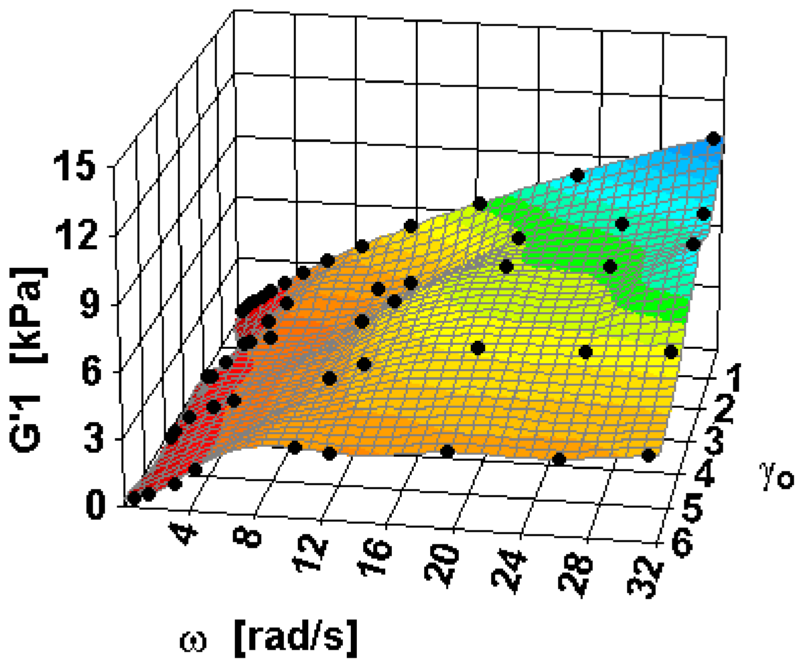
Fifth harmonic modulus G'5. Base asphalt, T = 27C



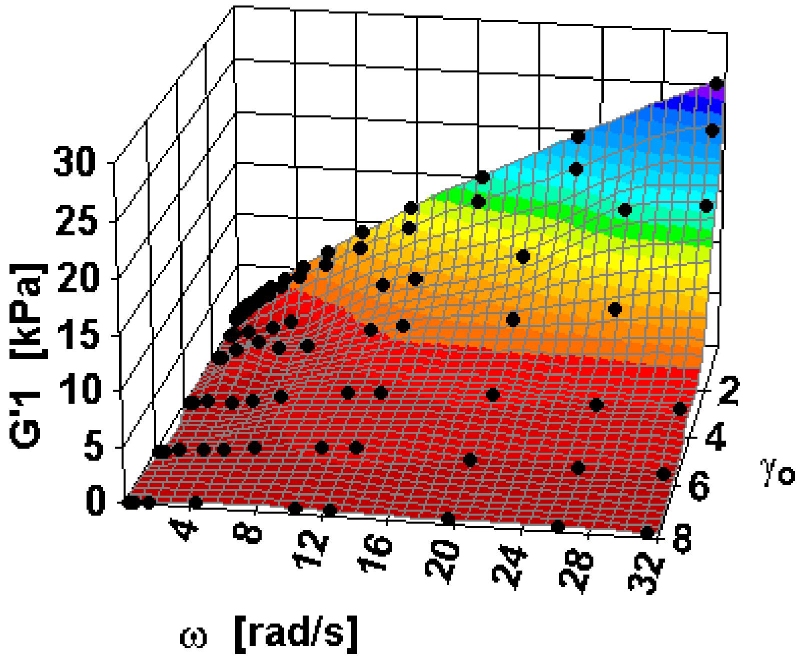
Fifth harmonic modulus G''5. Base asphalt, T = 27C.



First harmonic modulus G'1. Base asphalt with 6% SBS, T = 50C.



First harmonic modulus G'1. Base asphalt with 4% EVA, T = 34C.



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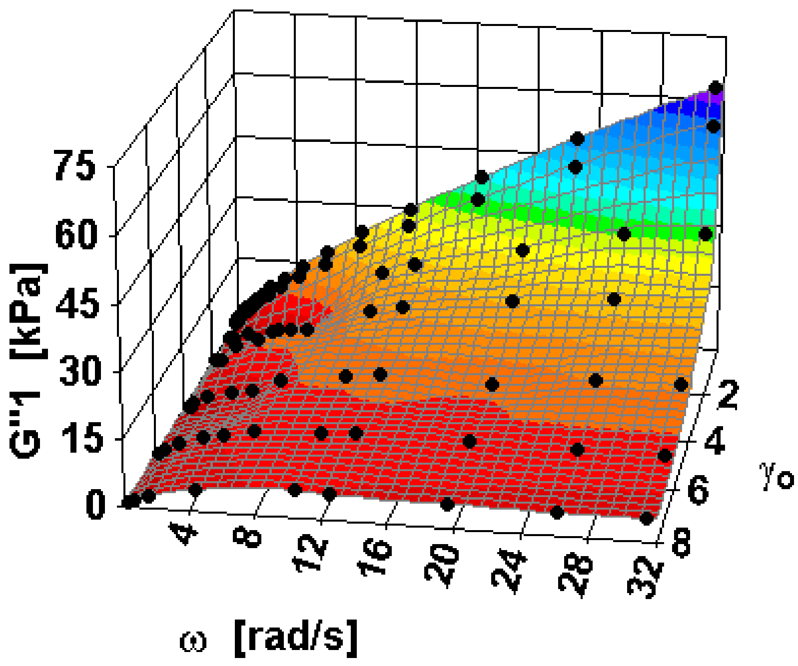
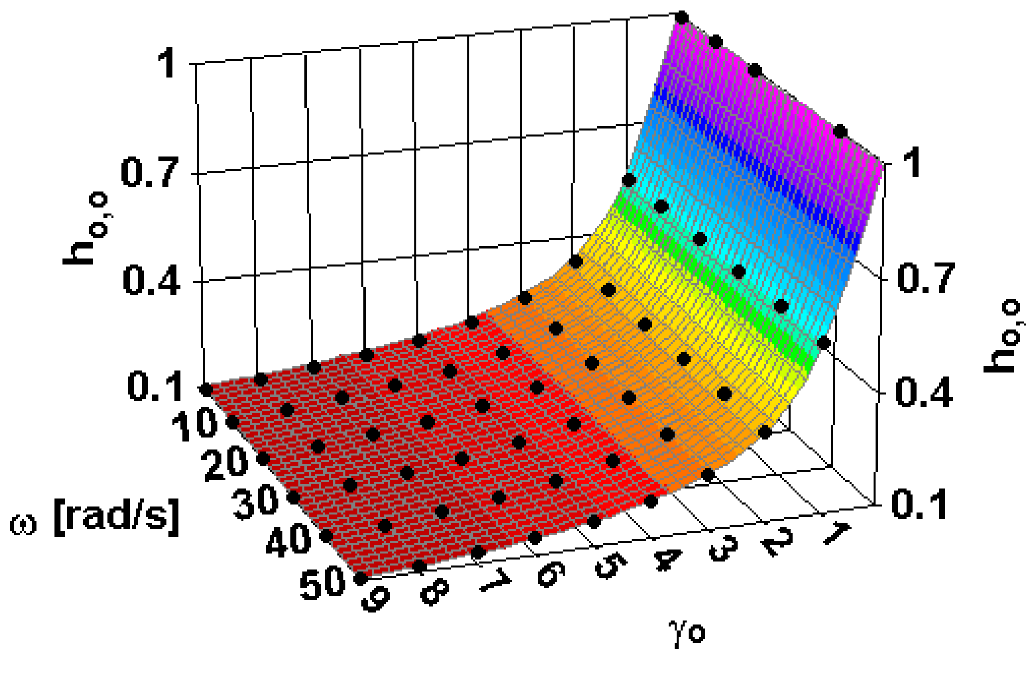
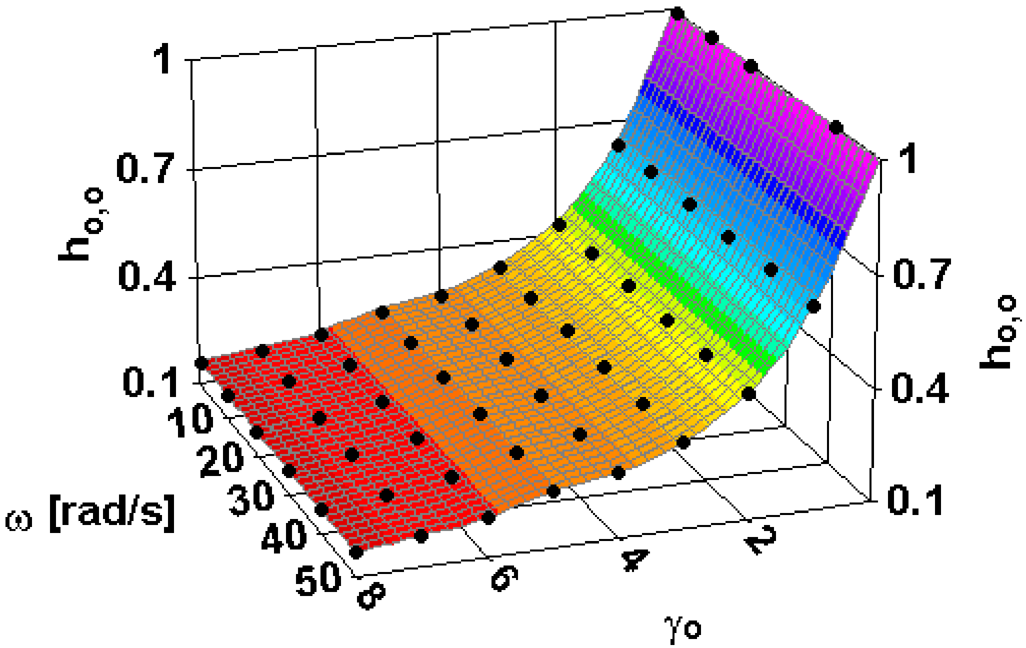


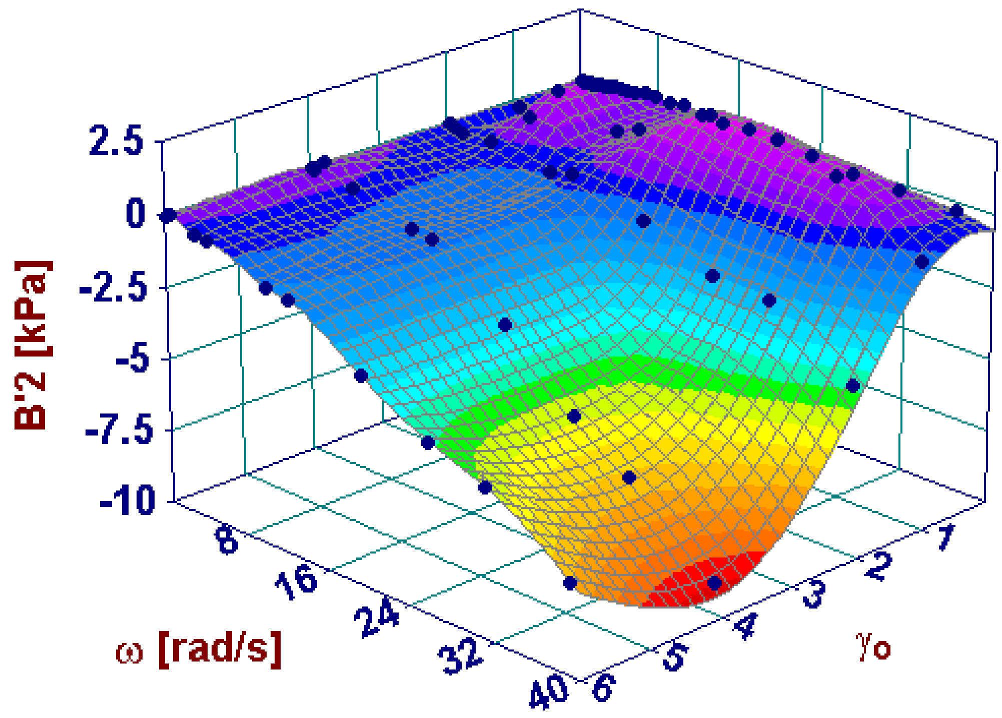
Exponential damping function - absolute term in double Fourier series.



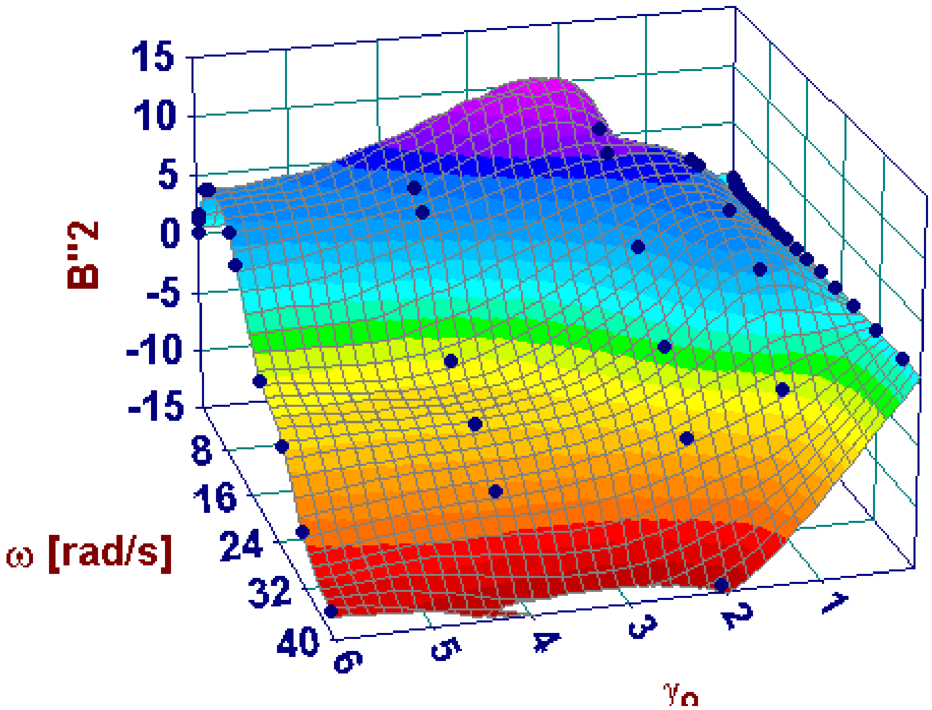
Fractional damping function of Soskey and Winter - absolute term in double Fourier series.



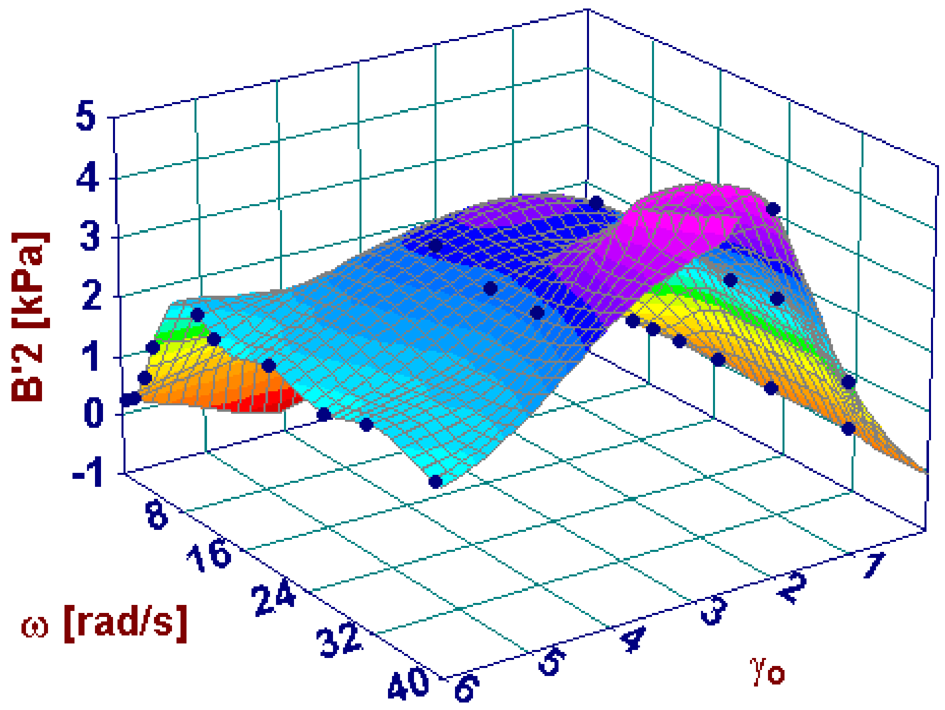
Exponential damping subtracted from G'1. Base asphalt, T = 27C.



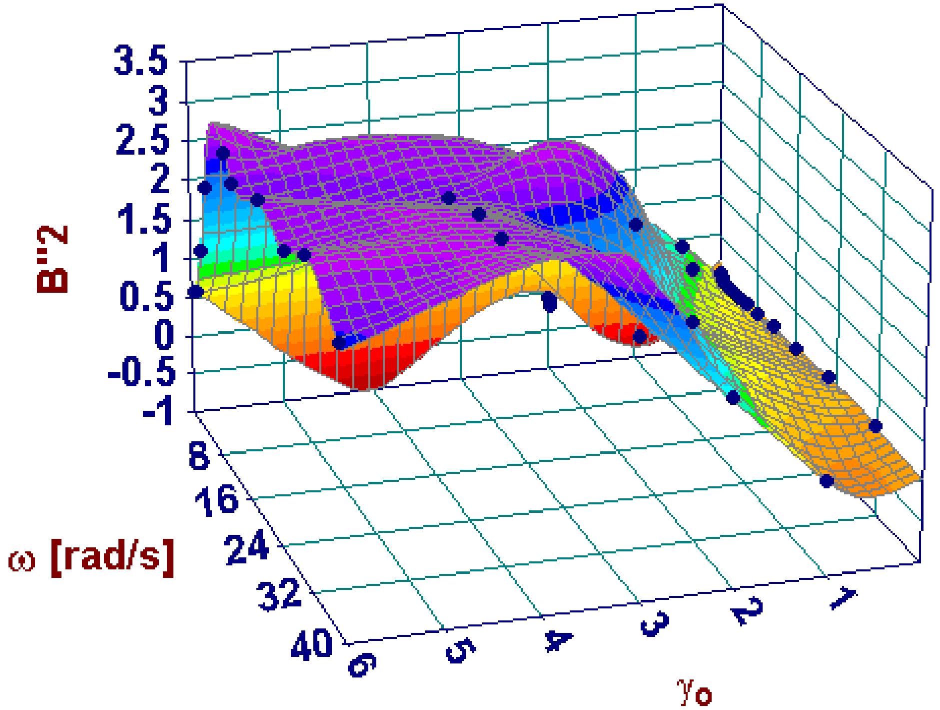
Exponential damping subtracted from G''1. Base asphalt, T = 27C.



Exponential damping subtracted from G'1. Base asphalt with 6% SBS, T = 50C.

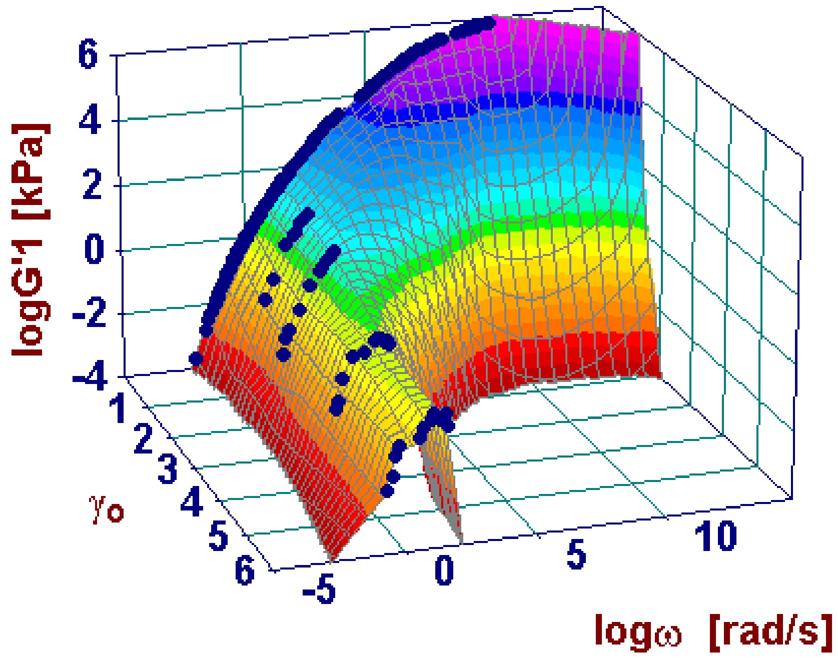


Exponential damping subtracted from G''1. Base asphalt with 6% SBS, T = 50C.



Comparison of linear viscoelastic master curve with LAOS.

First harmonic modulus G'1. Base asphalt, T = 27C.



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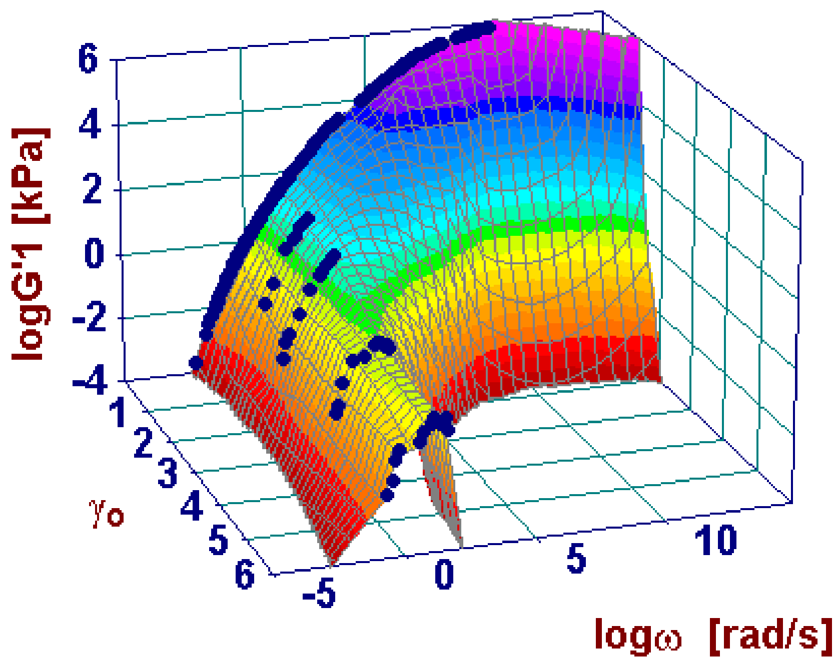

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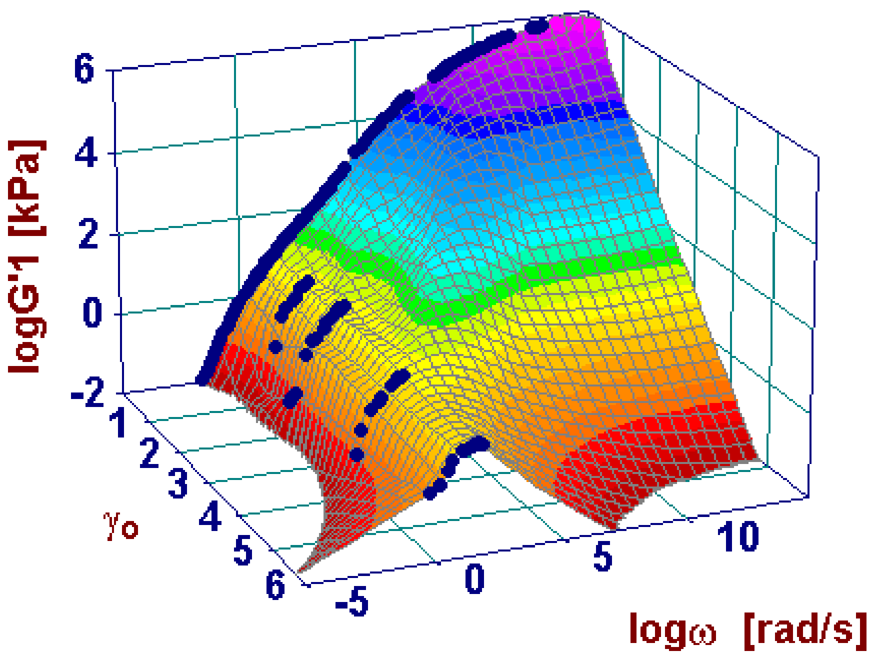

Comparison of linear viscoelastic master curve with LAOS. First harmonic modulus G''1. Base asphalt with

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